Economic Applications of Integral Calculus

* How Could you find the Total function from the marginal function?

Because the Process of integration is the opposite of differentiation, it should enable us to infer The Total function from agiven marginal function.

Example(1) if the marginal Cost (MC) of afirm is the following function of output $\boxed{C'(Q) = 2e^{0.2Q}}$ and if the fixed Cost is $C_F = 90$, find the Total Cost function $C(Q)$.

Solution

by integrating $C'(Q)$ with respect to Q, we find That:

$$\int 2e^{0.2Q}\,dQ = 2\frac{1}{0.2}e^{0.2Q} + C$$

$$\boxed{= 10e^{0.2Q} + C}$$

$$\boxed{\boxed{\text{Rule}}\ \int e^{ax}\,dx \text{ for } a \neq 0 = \frac{1}{a}e^{ax} + C}$$

Knowing that $C_F = 90$

that is when $Q =$ zero the Total Cost $= C_F$

$\Downarrow$

$$10e^{0.2(0)} + C = 90$$

$$10 + C = 90 \quad \therefore \boxed{C = 80}$$

$\boxed{\text{Example (2)}}$ if the marginal propensity to save (MPS) is the following function of income $\boxed{S'(y) = 0.3 - 0.1\, y^{-\frac{1}{2}}}$, and if the aggregate saving (S) is nil when income y is $\boxed{81}$, Find the saving function S(y).

$$S(y) = \int (0.3 - 0.1\, y^{-\frac{1}{2}})\, dy$$

$$= 0.3y - \frac{0.1}{\frac{1}{2}} y^{\frac{1}{2}} = \boxed{0.3y - 0.2y^{\frac{1}{2}} + \boxed{C}}$$

the value of $\boxed{C}$ Can be found as follows;

Sine S = zero when y = 81

$$0 = 0.3(81) - 0.2(81)^{\frac{1}{2}} + C$$

$$\therefore \boxed{C = -22.5}$$

the Saving function is

$$S(y) = 0.3y - 0.2y^{\frac{1}{2}} - 22.5$$

Investment and Capital Formation

Definition

Capital formation is the process of adding capital to a given stock of capital over time.

So, we can express capital stock as a function of time, $K(t)$ and use the derivative $\boxed{\frac{dk}{dt}}$ to denote the rate of capital formation.

Note that: the rate of capital formation at time (t) is identical with the rate of net investment flow at time (t) denoted by $I(t)$.

"the $I(t)$ is the derivative of $K(t)$"

that is, $\frac{dK}{dt} = I(t)$

and

$$\boxed{K(t) = \int I(t)\,dt = \int \frac{dk}{dt}\,dt = \int dk}$$

and

gross investment

$$I_g = I + \boxed{\delta K}$$

δ: the rate of depreciation

net investment

the rate of replacement investment

$\boxed{\text{Example 3}}$

Suppose that the net investment flow is described by the equation $I(t) = 3t^{\frac{1}{2}}$ and that the initial Capital stock, at time $t=0$, is $K(0)$. what is the Time Path of Capital K?

$\boxed{\text{Solution}}$

By integrating $I(t)$ w.r.t (t), we obtain:

$$K(t) = \int I(t)\,dt = \int 3t^{\frac{1}{2}}\,dt = \frac{3}{1.5}t^{1.5} + c$$

$$= \boxed{2t^{1.5} + c}$$

let $t=0$, we find that $K(0) = c$, therefore the Time Path of K is

$$\boxed{K(t) = 2t^{1.5} + K(0)}$$

Example 4

if the net investment is a Constant Flow at ~~I(t)~~.
$I(t) = 1000$ \$, what will be the total net investment (Capital formation) during a year from $t=0$ to $t=1$.

Solution

$$\int_0^1 I(t)\,dt = \int_0^1 1000\,dt = 1000t\Big]_0^1$$

$$\boxed{1000(1) - 1000(0) = 1000}$$

$$\boxed{\text{Rule}\quad \int_a^b I(t)\,dt = K(t)\Big]_a^b = K(b) - K(a)}$$

Example 5) if $I(t) = 3t^{\frac{1}{2}}$ "a non constant flow" what 'll be the capital formation during the interval [1, 4].

Solution)

by using the definite integral

$$\int_1^4 3t^{\frac{1}{2}}\,dt = 2t^{1.5}\Big]_1^4 = 2(4)^{1.5} - 2(1)^{1.5}$$

$$= 2(8) - 2$$

$$\boxed{= 16 - 2 = 14}$$

the present Value of Cash flow

$$A = V e^{-rt}$$

the present, the future values

$$\Pi = \int_0^y R e^{-rt} dt$$

discount rate

present value, future value

Example 6) What is the Present Value of a Continuous revenue flow Lasting For y years at a Constant rate of 'D' dollars per year and discounted at the rate of r per year?

$$\Pi = \int_0^y D e^{-rt} dt$$

$$= D \int_0^y e^{-rt} dt = D \cdot \frac{-1}{r} e^{-rt} \Big]_0^y$$

$$= \frac{-D}{r} e^{-r(y)} - \left[-\frac{D}{r}\right] = \frac{-D}{r}\left(e^{-ry} - 1\right)$$

$$\boxed{= \frac{D}{r}\left(1 - e^{-ry}\right)}$$

Present value of a Perpetual flow

Perpetual = Last forever

Ex.) the interest from a Perpetual bond or the revenue from an indestructible Capital asset such as Land.

the Present value Π will be $\Pi \int_0^{\infty} R(t) e^{-rt} dt$

"which is improper integral"

Example 6) Find the Present value of a Perpetual income stream flowing at a uniform rate of "D" dollars Per year, if the Continuous rate of discount is r.

Solution)

$$\Pi = \int_0^{\infty} D e^{-rt} dt = \lim_{y \to \infty} \int_0^{y} D e^{-rt} dt = \lim_{y \to \infty} \int_0^{y} D e^{-rt} \Big|_0^{y}$$

$$= \lim_{y \to \infty} \frac{D}{r} (1 - e^{-ry}) = \frac{D}{r}$$

thus: the Present value $= \dfrac{\text{Rate of revenue flow}}{\text{rate of discount}}$ →

this is also called

"Capitalization of an asset with a perpetual yield"

$$\boxed{\text{PV of Perpetuity} = \frac{D}{r}}$$