Mathematical Economics
Lec.4

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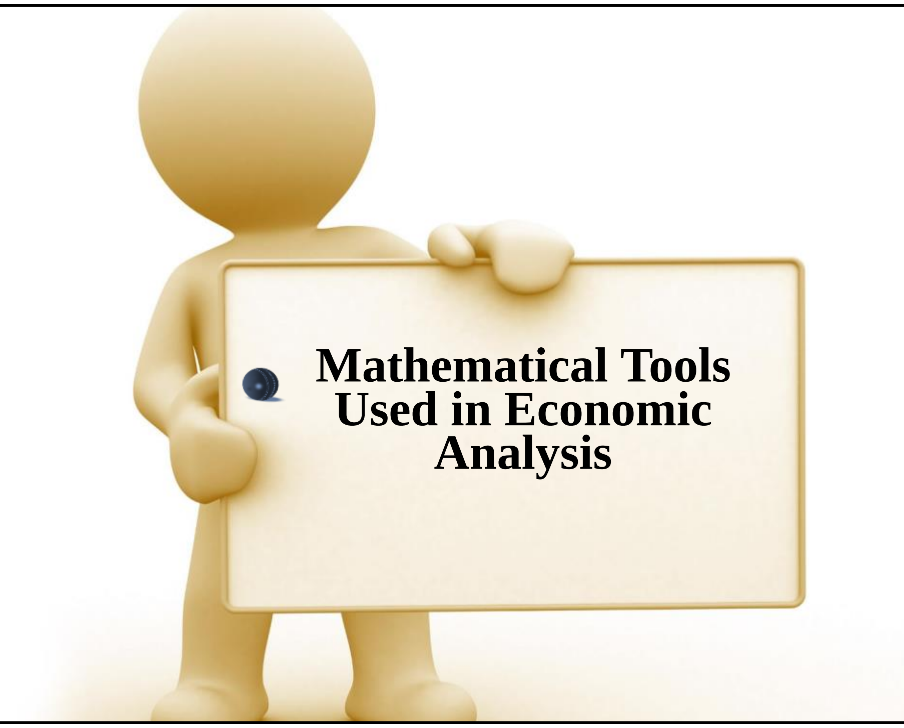
Mathematical Tools Used in Economic Analysis

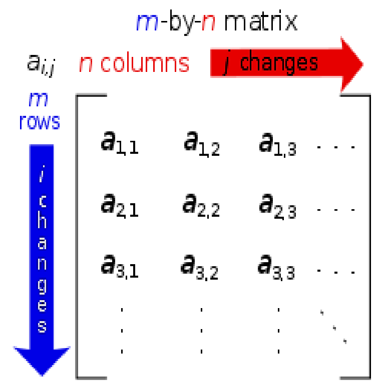


Mathematical Tools

Matrix Algebra	Differential calculus
Integral Calculus	Differential Equations

A matrix

- A matrix is a **rectangular array** of elements, i.e., numbers or any other objects.
- It is defined by number of **columns** and number of **rows**.
- We use matrices in mathematics and economics because often we need to deal with **several variables at once**.
- It is mainly used to solve system of linear equations.

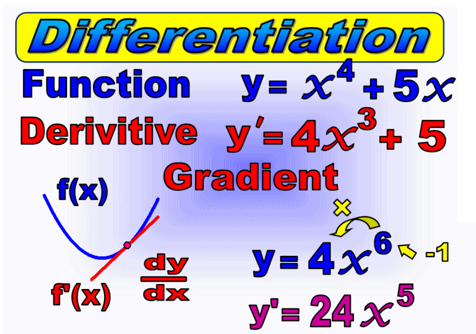


Differential Calculus

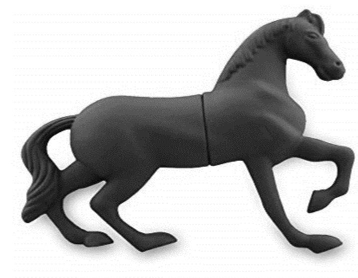
- Differential calculus is the branch of mathematics which studies **changes**. In economics, there are many problems containing two quantities such that the value of one depends upon the other. That is, a **variation** in the value of any ones produces a **variation** in the value of other.

- It is very useful tool to obtain the **rate** of **change** (the <u>speed by which the variable changes</u>). That is, derivatives is used to measure the small change in the **dependent variable** with reference to a very small change in the **independent variable**.

- Differentiation is a process of finding the **derivatives** of a **continuous function**.
- It is defined as the **limit value** of the ratio of the **change** in the **function** corresponding to **small change** in the independent variable, as the later tends to zero.

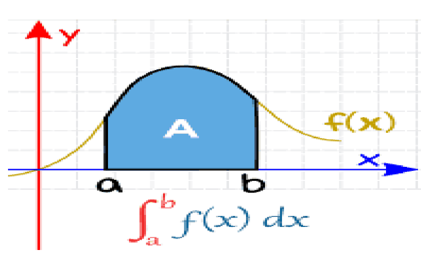


$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.$$



Integral Calculus

- While differential calculus allows us to find the rate at which a variable quantity changes, given its characteristic function. Integral calculus allows us to find the **function** defining a variable quantity, **<u>given its rate of change</u>**.

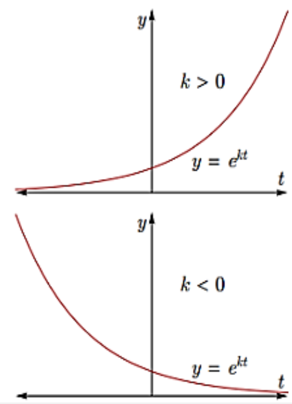


Differential Equations

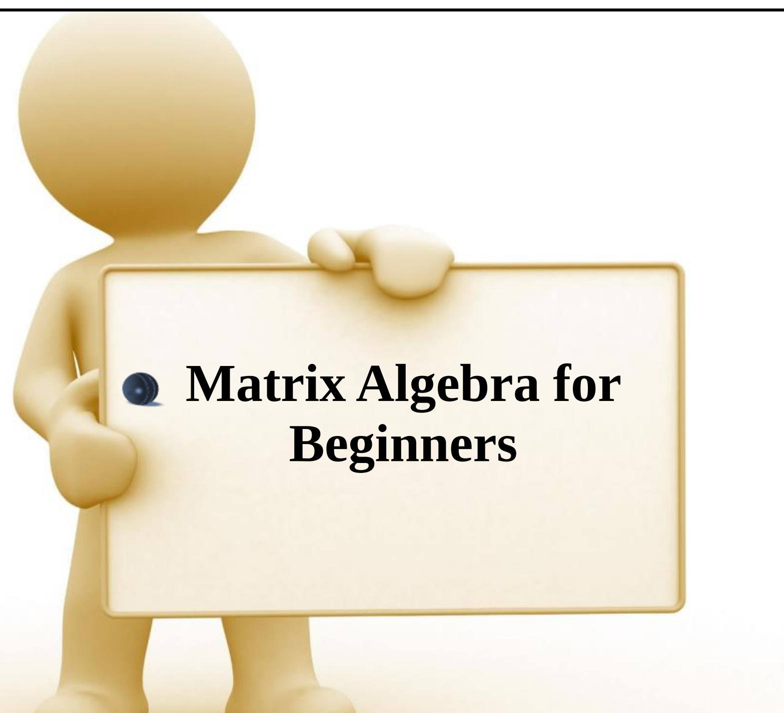
- Differential equation models are used in economics to model **dynamic models**. That is, the change in **time** according to some fixed rule. For such models, the **independent variable** is t (time) instead of x.
- The most common differential equation is:

  The Natural Growth Equation
- The natural growth equation is the differential equation of:

$$\frac{dy}{dt}=ky$$



Exponential Growth and Decay



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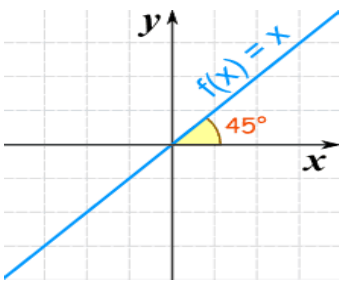
Q1) What is a linear equation?

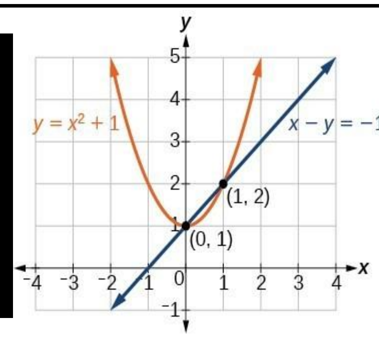
A linear Equation

- It is an equation of the form $a_1x_1 + a_2x_2 + \cdot\cdot\cdot + a_nx_n = b$,
- where $a_1, a_2, \ldots, a_n$ and b are constant real or complex numbers. The linear refers to the fact that the unknown quantities appear just as x and y, not as 1/x or y^3.

Linear equation



Non-Linear equation



Q2) What does “Systems of linear equations” mean?

Systems of linear equations

- A system of linear equations (or linear system) is a **finite collection** of linear equations in same variables. For instance, a linear system of m equations in n variables $x_1, x_2, . . . , x_n$ can be written as:

$$\mathbf{a_{11}x_1 + a_{12}x_2 + \cdots + {}_{a1nxn} = b_1}$$
$$\mathbf{a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = {}_{b2}}$$
$$\vdots$$
$$\mathbf{a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m}$$

Our attention should be given to "all **endogenous variables**"

- Note: The set of all solutions of a linear system is called the **solution set of the system**.

Theorem 1

- Any system of linear equations has one of the following exclusive conclusions: **(a)** No solution. **(b)** Unique solution. **(c)** Infinitely many solutions.

Q3) What is the main purpose of matrices?

Matrices

- Matrices first arose from trying to solve systems of linear equations.
- A matrix is any rectangular array of numbers. If the array has **n** rows and **m** columns, then it is an **n** × **m** matrix.

Note: The numbers n and m are called the dimensions of the matrix

We will usually denote matrices with capital letters, like **A**, **B**, etc, although we will sometimes use lower case letters for one dimensional matrices (a vector) (ie: 1 × m or n × 1 matrices).

Note: One dimensional matrices are often called vector

Note: - Row vector is a n × 1 matrix
- column vector is a 1 × m matrix

We will use the notation A_{ij} to refer to the number in the i-th row and j-th column.

Example of using matrices in Economics

→ **Example 1: Two markets equilibrium Model**

→ **Example 2: Expenditure model of national income**

Example 1: Two markets equilibrium Model

- By Assuming perfectly competitive market: that is, both buyers and sellers are price-takers. Two goods (coffee and tea).
- Two goods are substitutable (not complementary).
- Each producer can produce only one good (short-run).

Question

How can we find the equilibrium prices and quantities for multiple market models?

Two Markets

In market-1: we have the following equations:	In market-2: we have the following equations:
$Q_{d1} = 10 - 2P_1 + P_{2;}$ $Q_{s1} = -2 + 3P_1;$ $Q_{d1} = Q_{s1}$	$Q_{d2} = 15 + P_1 - P_{2;}$ $Q_{s2} = -1 + 2P_2;$ $Q_{d2} = Q_{s2}$

In market-1	In market-2
✓ Demand for good 1, $Q_{d1} = f(P_1, P_2)$ ✓ The coefficient of P_1 is negative due to the law of demand. ✓ The coefficient of P_2 is positive due to the fact that the two goods are substitutes. ✓ Producer produces only good-1	✓ Demand for good 2, $Q_{d2} = f(P_1, P_2)$ ✓ The coefficient of P_2 is negative due to the law of demand. ✓ The coefficient of P_1 is positive due to the fact that the two goods are substitutes. ✓ Producer produces only good-2

Question

How can we find the equilibrium prices and quantities for multiple market models?

Solution:

Step1: at equilibrium, equations can be rewritten as the following

In Market 1	In Market 2
$Q_1{}^* = 10 - 2P_1^* + P_2^*;$ $Q_1{}^* = -2 + 3P_1^*;$	$Q_2{}^* = 15 + P_1^* - P_2^*;$ $Q_2{}^* = -1 + 2P_2^*;$

Step2: Using Matrices:

$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 1 & 0 & -3 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix} * \begin{bmatrix} {Q_1}^* \\ {Q_2}^* \\ P_1^* \\ P_2^* \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 15 \\ -1 \end{bmatrix}$$

All Coefficients of Endogenous Variables

All Endogenous Variables

All Constants

Or $AX = B$

Coefficient matrix * Endogenous Variable Vector= Constant Vector

Note: The above matrix is a representation of the following system of linear equations:

$1xQ_1{}^* + 0xQ_2{}^* + 2P_1^* - 1xP_2^* = 10$ ➔Eq (1)

$1xQ_1{}^* + 0xQ_2{}^* - 3P_1^* + 0xP_2^* = -2$ ➔Eq (2)

$0xQ_1{}^* + 1xQ_2{}^* - 1xP_1^* + 1xP_2^* = 15$ ➔Eq (3)

$0xQ_1{}^* + 1xQ_2{}^* + 0xP_1^* - 2P_2^* = -1$ ➔Eq (4)

Step 3: The Solution

$$\begin{bmatrix} {Q_1}^* \\ {Q_2}^* \\ P_1^* \\ P_2^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 1 & 0 & -3 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix}^{-1} * \begin{bmatrix} 10 \\ -2 \\ 15 \\ -1 \end{bmatrix}$$

Or $\boldsymbol{X = A^{-1}B}$

Endogenous Variable Vector= the inverse of Coefficient matrix * Constant Vector

Example 2: Expenditure model of national income

- If you have the following variables for an economy: Y = Income C = Consumption I = Investment G = Government expenditure.
- If you know that income is the summation of all expenditures by the government, the household, and the private sector; the consumption function is a function of income provided that there exists an autonomous consumption. If you know that consumption and income are endogenous; while investment and government expenditure are exogenous.

Question

What are the solved values of income and consumption?

Question

What are the solved values of income and consumption?

Solution:

Step1: Given the previous information, we can find that:

Y = C + I + G ➔Income Equation

C = a + bY ➔Consumption Equation

Step2: Using Matrices:

$$\begin{bmatrix} 1 & -1 \\ -b & 1 \end{bmatrix} * \begin{bmatrix} Y \\ C \end{bmatrix} = \begin{bmatrix} I + G \\ a \end{bmatrix}$$

All Coefficients of Endogenous Variables

All Endogenous Variables

All Exogenous Variables or Constants

Or $AX = B$

Coefficient matrix * Endogenous Variable Vector= Constant Vector

Note that: The above matrix is a representation of the following system of linear equations:

1xY - 1xC = I + G ➔Eq (1)
-bY + 1xC = a ➔Eq (2)

Step 3: The Solution

$$\begin{bmatrix} Y \\ C \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -b1 & \end{bmatrix}^{-1} * \begin{bmatrix} I + G \\ a \end{bmatrix}$$

Or $\boldsymbol{X = A^{-1}B}$

Endogenous Variable Vector= the inverse of Coefficient matrix * Constant Vector

Thank you