Mathematical Economics
Lec.2

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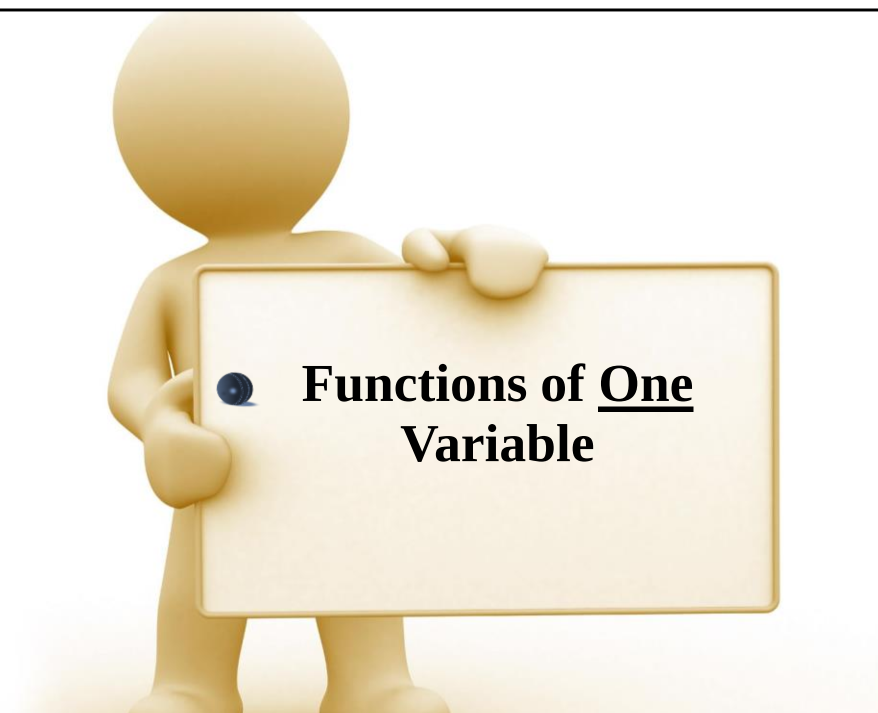
Mathematics Used in Microeconomics



Functions of One Variable

Variables

- The **basic elements** of algebra, usually called X, Y, and so on, that may be given any numerical value in an equation.

Functional Notation

- A way of **denoting** the fact that the value taken on by one variable (**Y**) depends on the value taken on by some other variable (**X**) or set of variables

$$Y = f(X)$$

Vector

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Independent and Dependent Variables

Independent Variable

- In an algebraic equation, a variable that is **unaffected** by the **action** of another variable and **may be assigned any value**

Dependent Variable

- In algebra, a variable whose value is **determined by** another **variable** or set of **variables**

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Two Possible Forms of Functional Relationships

- Y is a **linear function** of X
  - Example: Y = 3 + 2X
- Y is a **nonlinear function** of X
  - This includes X raised to powers other than 1
  - Example: a quadratic function $Y = -X^2 + 15X$

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Values of X and Y for Linear and Quadratic Functions

Linear Function		Quadratic Function	
x	Y = *f(X)* = 3 + 2X	x	Y = *f(X)* = $-X^2 + 15X$
-3	-3	-3	-54
-2	-1	-2	-34
-1	1	-1	-16
0	3	0	0
1	5	1	14
2	7	2	26
3	9	3	36
4	11	4	44
5	13	5	50
6	15	6	54

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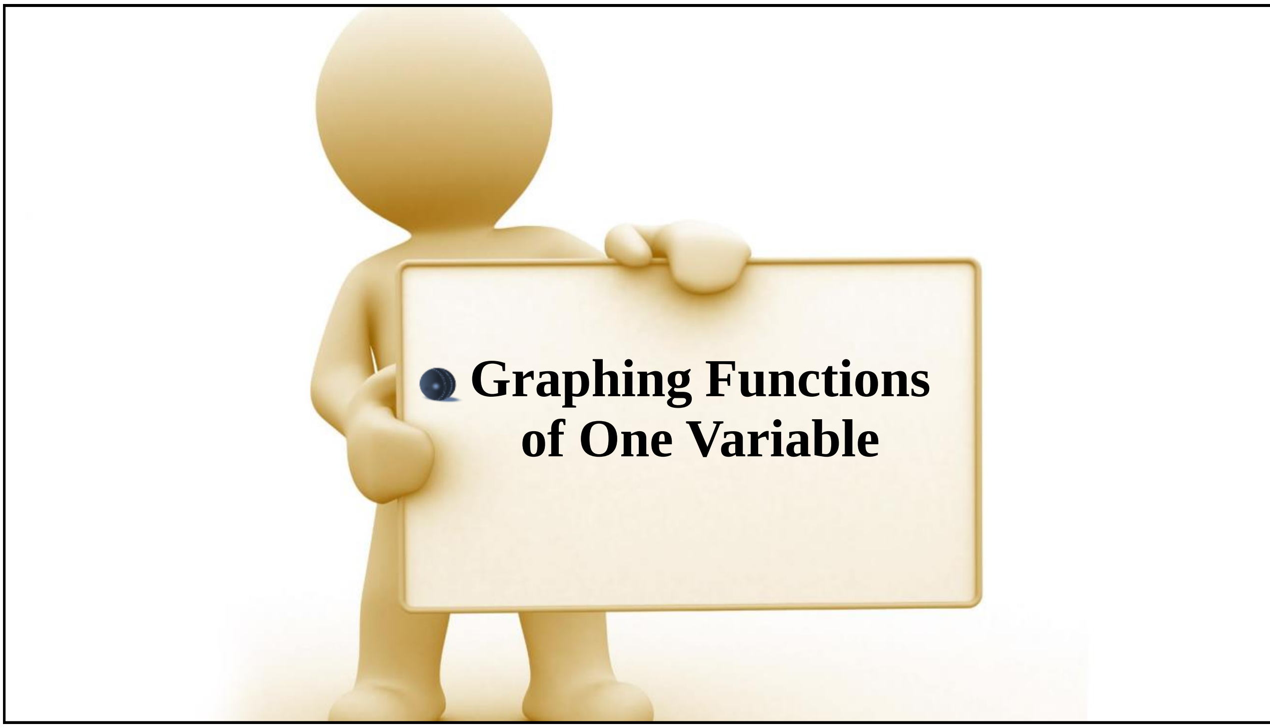

- **Graphs** are used to show the relationship between **two variables**
- Usually the dependent variable (Y) is shown on the vertical axis and the independent variable (X) is shown on the horizontal axis
  - However, on supply and demand curves, this approach is reversed (i.e., we have **Different Forms of Demand Functions**)

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Graphing a Linear Function

- A linear function is an equation that is represented by **a straight-line graph**
- Example, the linear function Y=3+2X
- Linear functions may take on both positive and negative values

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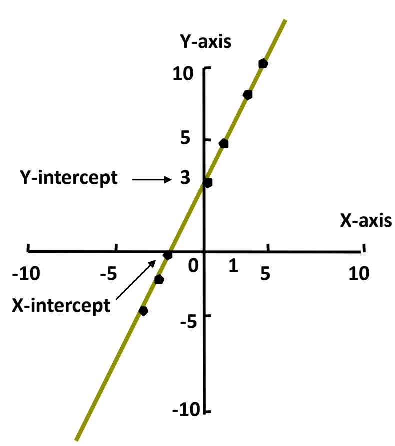
Graph of the Linear Function Y = 3 + 2X



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Intercept

- The general form of a linear equation is

$$Y = a + bX$$

- The Y-**intercept** is the value of Y when X equals 0
  - Using the general form, when X = 0, Y = a, so this is the intercept of the equation

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Slopes

- The **slope** of any straight line is the ratio of the change in Y (the dependent variable) to the change in X (the independent variable)
- The slope can be defined mathematically as

$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}} = \frac{\Delta Y}{\Delta X}$$

- where Δ means "change in"
- It is the direction of a line on a graph.

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Example,

- For the equation Y = 3 + 2X the slope equals 2 as can be seen in the previous figure by the dashed lines representing the changes in X and Y
- As X increases from 0 to 1, Y increases from 3 to 5

$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{5-3}{1-0} = 2$$

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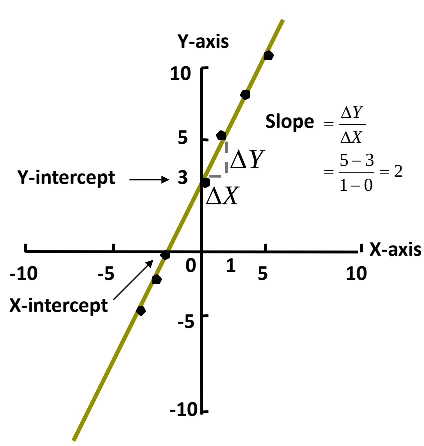
Graph of the Linear Function Y = 3 + 2X



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Important Notes about Slope:

- The **slope** is the **same** along a **straight line**.
- For the general form of the linear equation the slope equals **b (the parameter beside the x-variable)**
- The slope can be positive, negative, or zero
- If the slope is **zero**, the straight line is **horizontal** with Y = intercept.
- If the slope is **infinity**, the straight line is **vertical** with X=intercept.

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- The slope of a function depends on the **units** in which X and Y are **measured**:
  - If the **independent** variable in the equation Y = 3 + 2**X** is **income** and is measured in **hundreds** of **dollars**, a $100 increase would result in 2 more units of Y
  - If the same relationship was modeled but with X measured in single **dollars**, the equation would be Y = 3 + .02 **X** and the slope would equal .02, ($1increase would result in .02 more units of Y)

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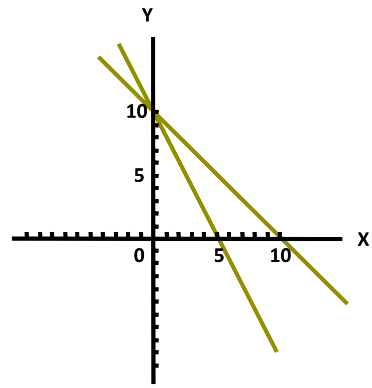
- A **change** in the **slope** of a function will cause **rotation** of the function **without changing the intercept**:



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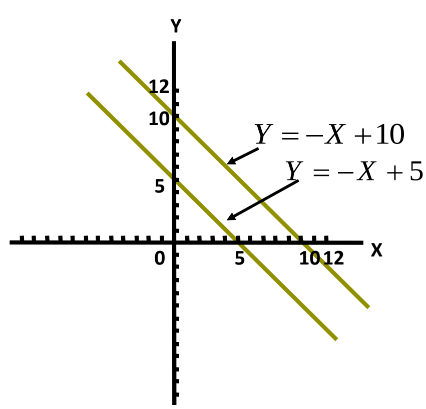
- When the **slope** is **held constant** but the **intercept** is **changed** in a linear function, this results in **parallel shifts** in the function:



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- Example of the nonlinear function $Y = -X^2 + 15X$
- As the graph shows, the slope of the line is not constant but, in this case, diminishes as X increases
- This results in a concave graph which could reflect the principle of diminishing returns

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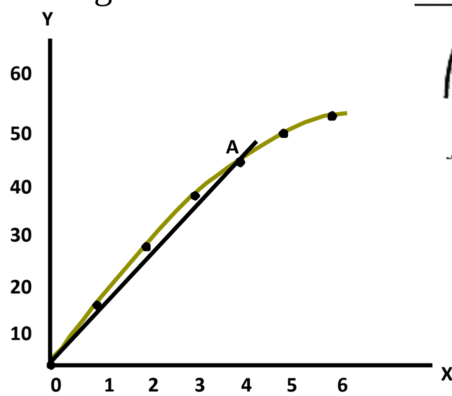
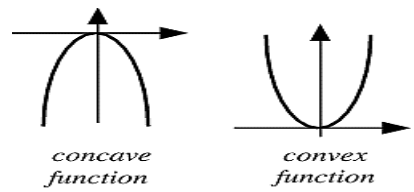
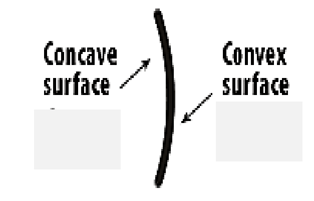
- Example of **the nonlinear function** Y = -X^2 + 15X
- As the graph shows, the slope of the line is **not constant** but, in this case, **diminishes** as **<u>X increases</u>**
- This results in a **<u>concave graph</u>** which could reflect the principle of diminishing returns







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Important Notes about a Nonlinear Function

- The graph of a nonlinear function is not a straight line
- Therefore it does not have the same slope at every point
- The slope of a nonlinear function at a particular point is defined as the slope of the straight line that is tangent to the function at that point.

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Marginal Effects

- The **marginal effect** is the change in Y brought about by one unit change in X at a particular value of X (Also the **slope** of the **function**)
- For a linear function this will be **constant**, but for a nonlinear function it will **vary** from point to point

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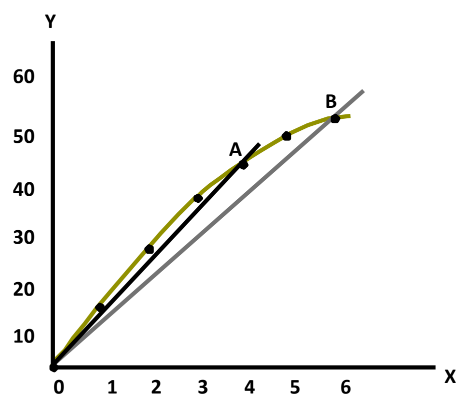
Average Effects

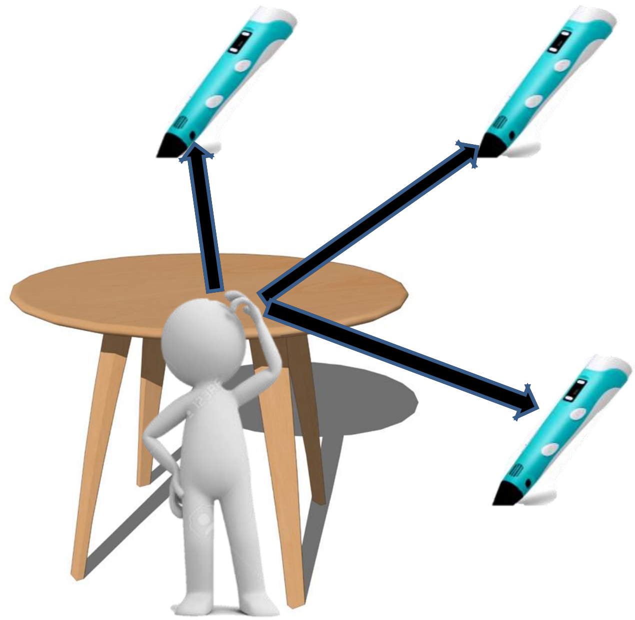
- The **average effect** is the ratio of **Y** to **X** at a particular value of X (the slope of a ray to a point)
- In the following figure, the ray that goes through A lies about the ray that goes through B indicating a higher average value at A than at B



A vector

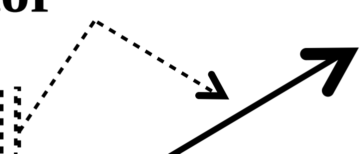
A vector

You don't know the location of the pen on the table; even if you know that it is at 1 arm distance away from you.



A vector

This is a vector



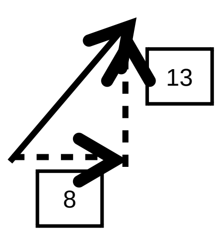
A vector has **magnitude** (size) and **direction**

- ✓ The length of the line shows its **magnitude**
- ✓ The arrowhead point shows the direction

(Head of the Vector)

For example:
A vector a = (8,13)

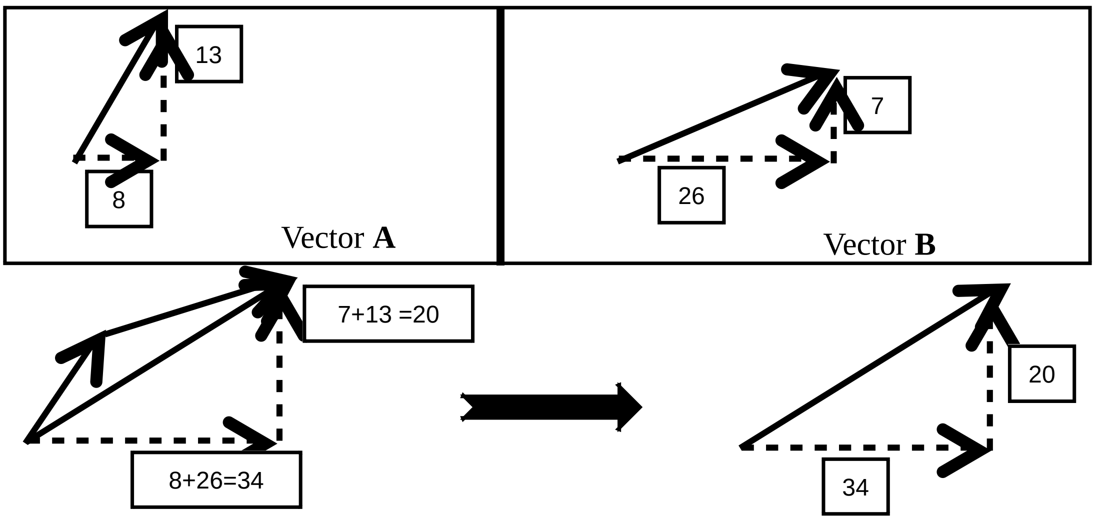
(Tail of the Vector)



- We can add **two vectors** by joining them **head-to-tail**

For example:

If we have the following two vectors: vector **A** and vector **B**

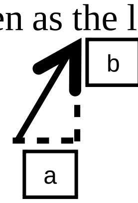


Symbolically,

- Vector **A**=(8,13) & vector **B**=(26,7),
- Create vector C = A+B?
- **C**= (8,13) + (26,7) = (8+26,13+7) = (34,20)

Notation of Vectors

- ✓ A vector is often written in **bold**, like **A** or **B**.
- ✓ A vector can also be written as the letters of its head and tail with an arrow above it, like A=$\overrightarrow{\boldsymbol{ab}}$



If Vector **A**=(12,2) & vector **B**=(4,5)

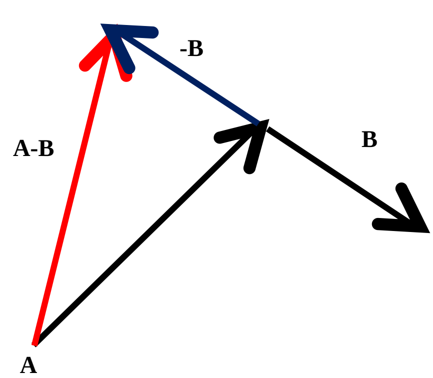
- Q) Create vector **C** by subtracting **B** = (4,5) from **A** = (12,2) ?

Steps:

1. we reverse the direction of the vector we want to subtract,
2. then add them as in addition

- C = A + −B
- C = (12,2) + −(4,5) = (12,2) + (−4,−5) = (12−4,2−5) = (8,−3)

Graphically,



Vector vs Scalar

- √ A <u>scalar</u> has <u>magnitude</u> (size) only.
- √ Scalar: just a number (like 7 or −0.32) ... definitely not a vector.
- √ A <u>vector</u> has <u>magnitude</u> and <u>direction</u>, and is often written in **bold**, so we know it is not a scalar:

For example,

- √ <u>A vector</u>**: c** is a vector, it has magnitude and direction
- √ <u>A scalar</u>: C is just a value, like 3 or 12.4
- √k**b** is actually the scalar k times the vector **b**.

The Different Forms of Demand Functions

Marshallian Demand (dX_1)

A function of the **price** of $\mathbf{X_1}$, the **price** of $\mathbf{X_2}$ (assuming two goods) and the **level of income** or wealth (**m**)	$X^*=dX_1(P_{X1}, P_{X2}, m)$

Hicksian Demand (hX_1)

A function of the **price** of $\mathbf{X_1}$, the **price** of $\mathbf{X_2}$ (assuming two goods) and the **level of utility** we opt for (U)	$X^*=hX_1(P_{X1},P_{X2},U)$

Thank you