Mathematical Economics
Lec.1

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Contents

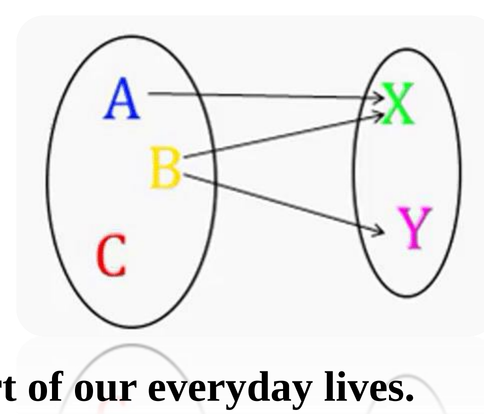
Introduction

Fundamental Methods of **Mathematical Economics**

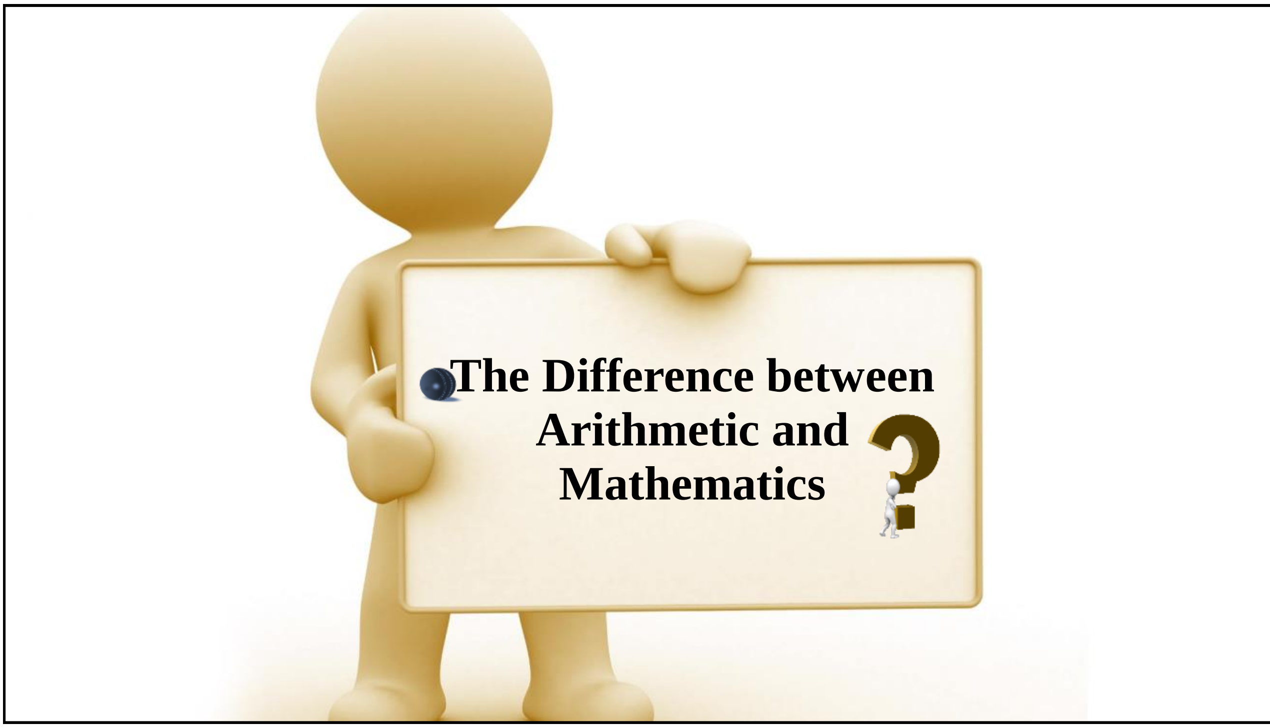
Mathematics: what?

Mathematicians are smart; but they are sophisticated (from another world)

- √ **Uses of Math:**
  - √ Simulation
  - √ Forecasting
  - √ Data Mining
  - √ Networking
  - √ Modeling



Mathematics is an integral part of our everyday lives.

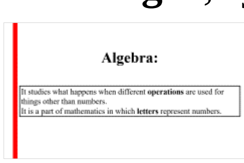


Arithmetic

(1) the branch of mathematics that deals with **addition**, **subtraction**, **multiplication**, and **division**,
(2) the use of **numbers** in <u>*calculations*</u>

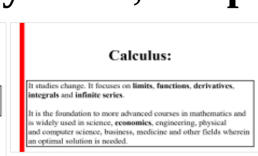
Mathematics

(1) the study of the **relationships** among numbers, shapes, and quantities,
(2) it uses **signs**, **symbols**, and **proofs** and includes **……………..**



Algebra:

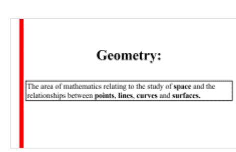
It studies what happens when different **operations** are used for things other than numbers.
It is a part of mathematics in which **letters** represent numbers.



Calculus:

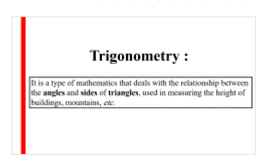
It studies change. It focuses on **limits**, **functions**, **derivatives**, **integrals** and **infinite series**.

It is the foundation to more advanced courses in mathematics and is widely used in science, **economics**, engineering, physical and computer science, business, medicine and other fields wherein an optimal solution is needed.



Geometry:

The area of mathematics relating to the study of **space** and the relationships between **points**, **lines**, **curves** and **surfaces.**



Trigonometry :

It is a type of mathematics that deals with the relationship between the **angles** and **sides** of **triangles**, used in measuring the height of buildings, mountains, *etc.*

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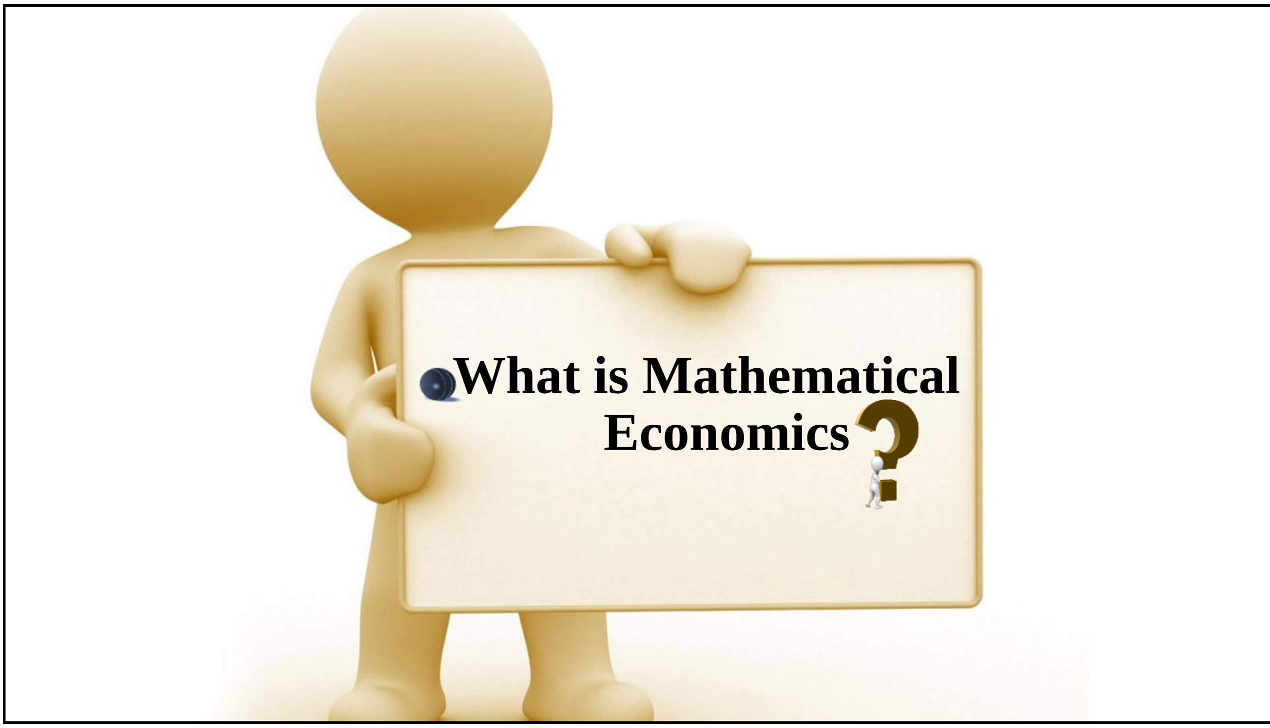
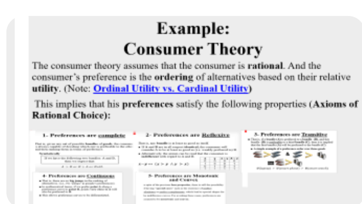


Is it a distinct branch of 'Economics'?

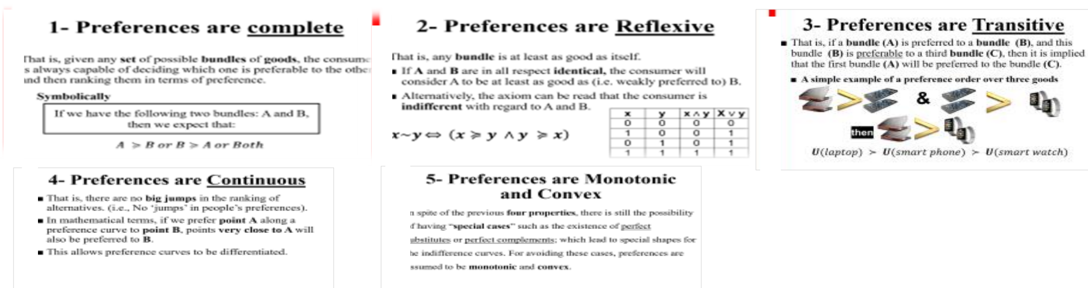
Rather It is an **approach** to **economic analysis**,

in which the economist makes use of **mathematical symbols** in the statements of the problem and also draws upon known mathematical theorems to aid in reasoning.



Example: Consumer Theory

- The consumer theory assumes that the consumer is **rational**. And the consumer's preference is the **ordering** of alternatives based on their relative **utility**. (Note: **Ordinal Utility vs. Cardinal Utility**)
- This implies that his **preferences** satisfy the following properties (**Axioms of Rational Choice):**



1- Preferences are <u>complete</u>

That is, given any **set** of possible **bundles** of **goods**, the consumer is always capable of deciding which one is preferable to the others and then ranking them in terms of preference.

Symbolically

If we have the following two bundles: A and B, then we expect that:

$$A \succcurlyeq B \text{ or } B \succcurlyeq A \text{ or Both}$$

2- Preferences are <u>Reflexive</u>

That is, any **bundle** is at least as good as itself.

- If **A** and **B** are in all respect **identical,** the consumer will consider A to be at least as good as (i.e. weakly preferred to) B.
- Alternatively, the axiom can be read that the consumer is **indifferent** with regard to A and B.

$$x \sim y \Leftrightarrow (x \succcurlyeq y \wedge y \succcurlyeq x)$$

x	y	x ∧ y	X ∨ y
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	1

3- Preferences are <u>Transitive</u>

- That is, if a **bundle (A)** is preferred to a **bundle (B)**, and this bundle **(B)** is <u>preferable</u> to a third **bundle (C)**, then it is implied that the first bundle **(A)** will be preferred to the bundle **(C)**.
  - **A simple example of a preference order over three goods**

$$U(laptop) \succ U(smart\ phone) \succ U(smart\ watch)$$

If bundle **A** is **Strictly** preferred to bundle **B**, then **A** is **Strictly** preferred to **C**.

Symbolically,

$$If\ A \succ B\ and\ B \succ C, then\ A \succ C$$

If bundle **A** is **Weakly** preferred to bundle **B**, then **A** is **Weakly** preferred to **C**.

Symbolically,

$$If\ A \succcurlyeq B\ and\ B \succcurlyeq C, then\ A \succcurlyeq C$$

4- Preferences are Continuous

- That is, there are no **big jumps** in the ranking of alternatives. (i.e., No ‘jumps’ in people’s preferences).
- In mathematical terms, if we prefer **point A** along a preference curve to **point B**, points **very close to A** will also be preferred to **B**.
- This allows preference curves to be differentiated.

5- Preferences are Monotonic and Convex

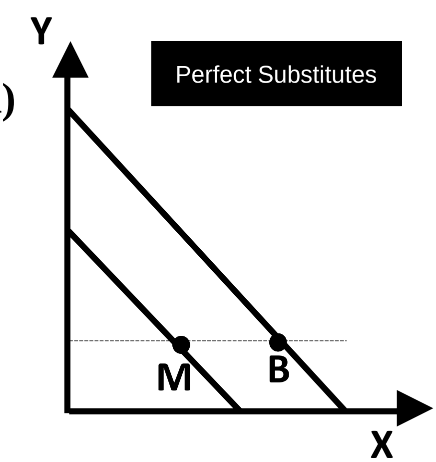
In spite of the previous **four properties**, there is still the possibility of having "**special cases**" such as the existence of perfect substitutes or perfect complements; which lead to special shapes for the indifference curves. For avoiding these cases, preferences are assumed to be **monotonic** and **convex**.

Monotonic:

- A **rational consumer** always prefers **more** of a commodity as it offers him a higher level of **satisfaction**.

Strict Monotonic Preference:

That is, given any set of two **bundles (A)** and **(B)**, if one of them (e.g., bundle **A**) contains more of one good, and **not less** in any other good, then the first bundle (A) will be preferred to the second (B).

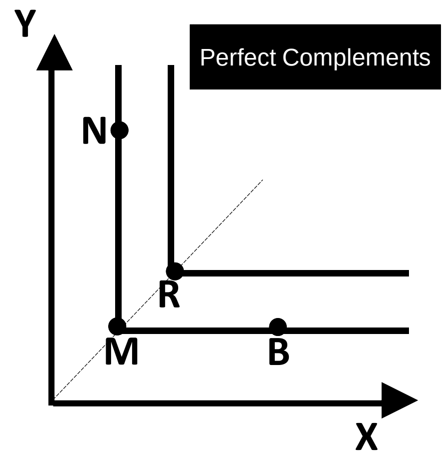


Symbolically,

If $X = (X_1, X_2)$ and $Y = (Y_1, Y_2)$, $X \gg Y$, $X1 > Y1 \wedge X2 = Y2 \Rightarrow X \succ Y$

Monotonic:

Weak Monotonic Preference:
That is, given any set of two bundles **(A)** and **(B)**, if one of them (e.g., bundle A) contains more of both goods, then the **first bundle (A)** will be preferred to **the second (B).**



Symbolically,

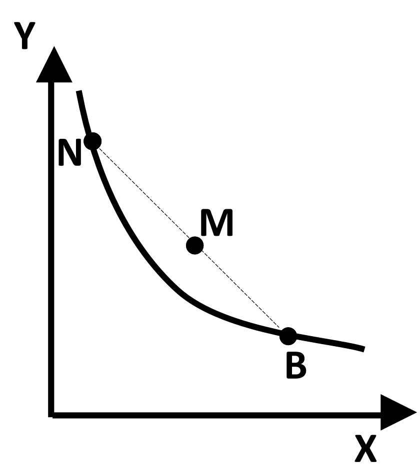
If X = (X_1,X_2) and Y = (Y_1,Y_2), X ⋙ Y, X1>Y1 ∧ X2>Y2 ⇒ X ≽ Y

Convex:
- That is, any **combination** of **two equally** preferable bundles will be more desirable than these **bundles**.
- That is, That is **average** is always preferred to the **extreme** bundles.

Strictly convex: If x is indifferent to y, then any **mixture** of **x** and y is strictly preferred to either

Symbolically,

$$if\ x \backsim y \Rightarrow \alpha x + (1-\alpha) y \succ x$$



Weekly convex: If x is indifferent to y, then any **mixture** of **x** and **y** is weakly preferred to either

Symbolically,

$$\boldsymbol{if\ x \backsim y \Rightarrow\ \alpha x + (1 - \alpha) y \succcurlyeq x}$$

Ordinal Utility vs. Cardinal Utility

- **<u>Ordinal Utility</u>:** An ordinal utility function, is a utility function where **differences** between U (x) and U (y) are **meaningless**.
- **<u>Cardinal Utility</u>:** A cardinal utility function is one in which **differences** between U (x) and U (y) are **meaningful** as they reflect the **intensity** of **preferences**.

Thank you