# Leaflet-Contents

# Leaflet- Definitions

## Equations' Elements

### A variable

It is something whose magnitude can change, i.e., something that can take on different values. e.g., price (p) , cost (c), revenues (R), and profits (π). It must be represented by a symbol instead of a specific number.

### Endogenous Variable

It is a variable that is originating from within. Its solution value is determined from the model.

### Exogenous Variable

It is a variable that is originating from without. Its solution values is determined by forces external to the model. E.g., $P_{0}$.

### Dependent Variable

a variable whose value depends on that of another.

### Independent Variable

a variable whose value doesn't depend on that of another.

### Constants

A magnitude that doesn't change and is therefore the antithesis (the opposite) of a variable.

### Coefficients

A constant that is joined to a variable. there are two types: symbolic coefficient (parameter/parametric constants) and numeric coefficient.

### Ordered Pairs

An ordered pair (a, b) is a pair of objects. The order in which the objects appear in the pair is significant: the ordered pair (a, b) is different from the ordered pair (b, a) unless a = b.

### Unordered Paris

An unordered pair {a, b} is a pair of objects. The order in which the objects appear in the pair is meaningless; thus, {a, b}={b, a}

## A Function

A process or a relation that associates each element x of a set X (the domain of the function) to a single element y of another set Y (the codomain of the function). A function is also called a mapping or transformation (i.e., a rule by which the set X is mapped (transformed) into the set Y. symbolically, $f: x \rightarrow Y$

## Domain

The set of all possible values that the independent variable(s) can take (i.e., the set of x-values that give rise to y-values).

## The Image of the x-Value

The y value into which an x value is mapped (i.e., the corresponding y value).

## Range of a Function

It refers to either the codomain or the image of the function. (i.e., the set of all values that the y variable will take)

## A Matrix

It is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. The members of the array are called the elements of the matrix. It provides a compact way of writing an equation system. It is applicable only to linear-equation systems.

## Linear Approximation for Nonlinear Relation

A nonlinear function can be approximated with a linear function in a certain operating point. A number of methods are used for linear approximation, among them is taking the logarithm on both sides of the nonlinear equation.

# Static or Equilibrium Analysis

## Equilibrium

It is a constellation of selected interrelated variables so adjusted to one another that no inherent tendency to change prevails in the model which they constitute. Thus, all selected variables in the model must simultaneously be in a state of rest (the balancing of the internal forces of the model); while, the external factors are assumed fixed. The equilibrium state does not necessarily represent a desirable or ideal state.

## Static Analysis (Equilibrium Analysis)

The main objective of such analysis is to find the values of endogenous variables that satisfy the equilibrium condition given specific values for the exogenous variables and/ or the parameters.

# Comparative Static Analysis

## Comparative Static Analysis

Comparative static analysis can be used to show economic equilibrium before and after a change in one or more variables without regard to the time required. In another meaning, it is used to compare between an **initial equilibrium state** and a **new equilibrium state**. Such analysis attempts to answer the following question: how does the equilibrium value of an endogenous variable change with respect to a change in the value of one of the exogenous variables and/or parameters? Thus, it aims at identifying the rate of change.

## The Derivative

The derivative (the limit) is a function derived from the primitive function and has the same explanatory variable as the primitive function. The derivative measures the rate of change.

## Marginal Analysis

Marginal analysis is the study of the rate of change of economic quantities.

## Continuity and Differentiability of a Function

A function q= g(v) is continuous at v = N, if and only if, the following conditions are met:

- The point N is in the domain of the function, that is the function g(N) is defined at v=N.
- The limit of the function as x approaches a exists

It should be noted that: the continuity of a function is a necessary condition for its

differentiability.

# Types of Mathematical Equations used in Economics

**Definitional Equation:** is one that defines a particular concept. It is not important which variables are considered independent or dependent. (the identical-equality sign ≡ is used). E.g., $Y \equiv C + I + X - N; \pi = R - C$

**Behavioral Equation:** is one that specifies the manner in which a variable behaves in response to changes in other variables. Generally, it is used to describe the general institutional setting of a model, including the technological (e.g., production function) and legal (e.g., tax structure) aspects.

**Conditional Equation (equilibrium condition):** an equation that describes the prerequisite for the attainment of equilibrium. (e.g., $Q_d = Q_s$)

# Types of Functions

## ■ Algebraic Functions

Any function expressed in terms of polynomials and/or roots of polynomials is algebraic functions.

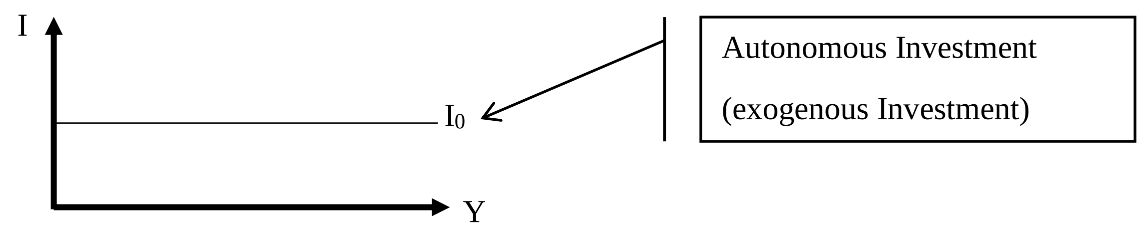
## ■ Polynomial (Multiterm) Function

A function such as a quadratic, a cubic, a quartic, and so on, involving only non-negative integer powers of x. A polynomial function of degree (n) for a single variable (x) is a function of the form:

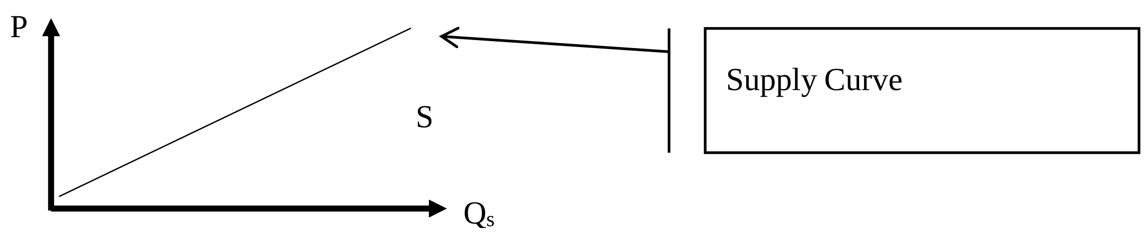
$y = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ ;

The degree of a polynomial is the highest power of x (i.e. n). accordingly, there are a number of cases:

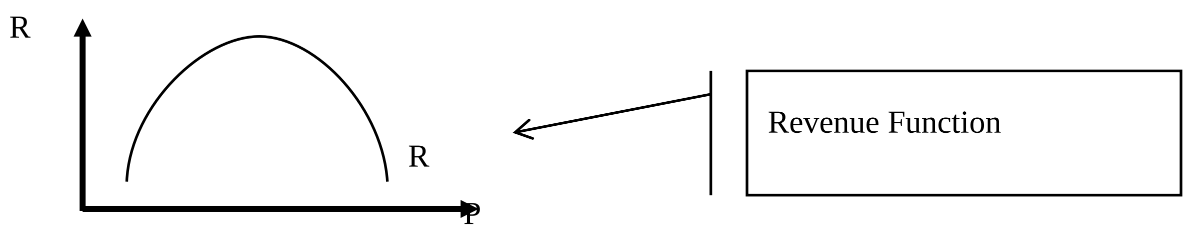
✓ If n=0: $y = a_0$ [**Constant Function**]



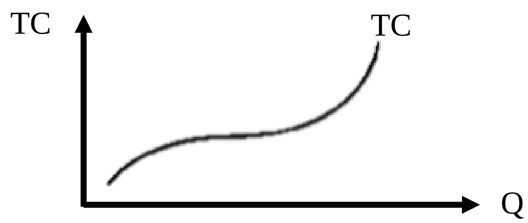

✓ If n=1: $y = a_0 + a_1x$ [**Linear Function**]




✓ If n=2: $y = a_0 + a_1x + a_2x^2$ [**Quadratic Function**]



✓ If n=3: $y = a_0 + a_1x + a_2x^2 + a_3x^3$ [**Cubic Function**]



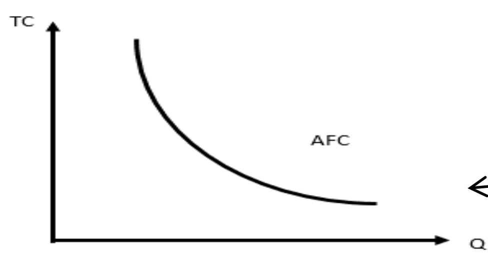

Long-Term Total Cost Function

## ■ Rational Function

Any function in which y is expressed as a ratio of two polynomials in the variable x. (e.g., $y = \frac{a}{x}$ or $xy = a$

Graphically, it can be represented as rectangular hyperbolic curve (e.g., average

fixed cost curve).

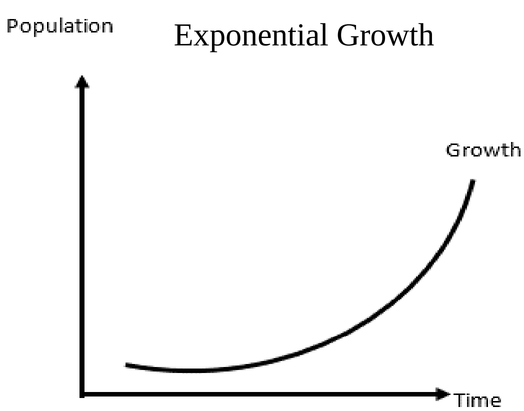



Average Fixed Cost Function

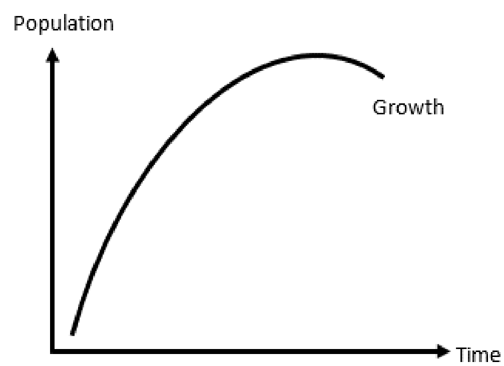
## ■ Non-Algebraic Functions

Any function in which the independent variable appears either in the exponent (exponential function) or as a logarithm (logarithmic function)

Exponential Growth




Logistic Growth



# Leaflet- Rules

## Matrix Operations

### Equality condition of Matrices

P51

$A = [a_{ij}] = B = [b_{ij}] \Leftrightarrow a_{ij} = b_{ij}\ for\ all\ values\ of\ i\ and\ j$

### Addition and Subtraction of Matrices

P51-52

Two **matrices** may be added or subtracted only if they have the same **dimension**; that is, they must have the same number of rows and columns.

If $\boldsymbol{A}_{2.2} = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$ $and$ $\boldsymbol{B}_{2.2} = \begin{bmatrix} b11 & b12 \\ b21 & b22 \end{bmatrix}$, then

$$\boldsymbol{A}_{2.2} + \boldsymbol{B}_{2.2} = \begin{bmatrix} a11 + b11 & a12 + b12 \\ a21 + b21 & a22 + b22 \end{bmatrix}$$

$$\boldsymbol{A}_{2.2} - \boldsymbol{B}_{2.2} = \begin{bmatrix} a11 - b11 & a12 - b12 \\ a21 - b21 & a22 - b22 \end{bmatrix}$$

$$\boldsymbol{A}_{2.2} + \boldsymbol{A}_{2.2} = \boldsymbol{2A}_{2.2}$$

**Rule**: When two matrices are compatible we add (or subtract) them by adding (or subtracting) the elements in corresponding positions.

### Scalar Multiplication

P52

Scalar multiplication is where a matrix is multiplied by a single number.

If $\boldsymbol{A}_{2.2} = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$ $and\ \omega\ is\ a\ scalar$, then

$$\omega * \boldsymbol{A}_{2.2} = \begin{bmatrix} \omega * a11 & \omega * a12 \\ \omega * a21 & \omega * a22 \end{bmatrix}$$

**Rule:** To multiply a matrix by a scalar (that is, a single number), we simply multiply each element in the matrix by this number.

### Multiplication of Matrices

P53-59

Matrix multiplication is where a matrix is multiplied by another matrix.

If $\boldsymbol{A}_{2.2} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{bmatrix}$ *and* $\boldsymbol{B}_{2.1} = \begin{bmatrix} \boldsymbol{b_{11}} \\ \boldsymbol{b_{21}} \end{bmatrix}$, then

$$\boldsymbol{A}_{2.2} * \boldsymbol{B}_{2.1} = \begin{bmatrix} \boldsymbol{a_{11}} * \boldsymbol{b_{11}} + \boldsymbol{a_{12}} * \boldsymbol{b_{21}} \\ \boldsymbol{a_{21}} * \boldsymbol{b_{11}} + \boldsymbol{a_{22}} * \boldsymbol{b_{21}} \end{bmatrix}_{2.1}$$

**Rule**: Two matrices can only be multiplied together if the number of columns in the first matrix is the same as the number of rows in the second.

## ■ Multiplication of Vectors

P59-60

If $\boldsymbol{u}_{2.1} = \begin{bmatrix} \boldsymbol{a_{11}} \\ \boldsymbol{a_{21}} \end{bmatrix}$ *and* $\acute{\boldsymbol{v}}_{1.3} = [b_{11} \;\; b_{12} \;\; b_{13}]$, then

$$\boldsymbol{u}_{2.1} * \acute{v}_{1.3} = \boldsymbol{u\acute{v}}_{2.3} = \begin{bmatrix} \boldsymbol{a_{11}} * \boldsymbol{b_{11}} + \boldsymbol{a_{11}} * \boldsymbol{b_{12}} + \boldsymbol{a_{11}} * \boldsymbol{b_{13}} \\ \boldsymbol{a_{21}} * \boldsymbol{b_{11}} + \boldsymbol{a_{21}} * \boldsymbol{b_{12}} + \boldsymbol{a_{21}} * \boldsymbol{b_{13}} \end{bmatrix}_{2.3}$$

Given a row vector Ú, the $\acute{U}U$ = scalar → it is **the sum of squares** of the elements $U_j$

$$\acute{\mathrm{U}}\,\mathrm{U} = \mathrm{U}_1^2 + \mathrm{U}_2^2 + \cdots \ldots + \mathrm{U}_n^2 = \sum_{j=1}^{n} \mathrm{U}_j^2$$

**Rule**: An m x 1 column vector u, and 1 x n row vector $\acute{v}$, yield a product matrix u$\acute{v}$ of dimension m x n.

## ■ Linear Dependence of Vectors

P62-63

vectors $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots, \boldsymbol{v_k}\}$ is linearly dependent ⟺ $\boldsymbol{r_1} \boldsymbol{v_1} + \boldsymbol{r_2} \boldsymbol{v_2} + \cdots + \boldsymbol{r_k} \boldsymbol{v_k} = 0$

**Rule**: the set of vectors $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \ldots, \boldsymbol{v_k}\}$ is linearly dependent if and only if any one of them can be expressed as a linear combination of the remaining vectors; otherwise they are linearly independent.

## ■ Identity Matrix (I)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Rule:**

- **Identity Matrix** is a square matrix in which all the elements of the principal diagonal are ones and all other elements are zeros.
- If A is a square matrix, then IA = A = AI

## ■ A zero Matrix (0)

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

**Rule: A zero matrix** is a matrix with all zero entries

## ■ Diagonal Matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

**Rule: Diagonal Matrix** is a square matrix whose off-diagonal entries are all zero.

## ■ Transposes of a Matrix

$P_{73\text{-}74}$

If $\boldsymbol{A_{2.3}} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & \boldsymbol{a_{13}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} \end{bmatrix}$**, then**

$$\boldsymbol{\acute{A}_{3.2}} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{21}} \\ \boldsymbol{a_{12}} & \boldsymbol{a_{22}} \\ \boldsymbol{a_{13}} & \boldsymbol{a_{23}} \end{bmatrix}$$

**Notes**:

- The transpose of a matrix A is denoted $A^T$ or $A'$. The transpose of a matrix exchanges the rows and columns. The $i^{th}$ column becomes the $j^{th}$ row. Or the $a_{ij}$ entry becomes the $a_{ji}$ entry.
- You should note that the change in the matrix's dimension. Thus, if a matrix A is m x n, then its transpose $A'$ must be n x m.
- An n x n square matrix possesses a transpose with the same dimension.
- Symmetric Matrices are square matrices that satisfy $A = A^T$

Rules:
$(AB)^T = B^T A^T$
$(A^t)^t = A$
$(A+B)^t = A^t + B^t$

## ■ Inverse of a Matrix

$P_{75\text{-}77}$

Let A be a square matrix. We denote the inverse by $A^{-1}$, and it have to have the following properties:

- $AA^{-1} = I$
- $A^{-1}A = I$.

**Notes:**

- Not every matrix has an inverse: squareness is a necessary condition, but not a sufficient condition. Thus, the notion of an inverse matrix only applies to square matrices.
- Requirements to have an Inverse: **(1)** The matrix must be square; **(2)** The determinant of the matrix must not be zero.
- If a square matrix A has an inverse, A is said to be nonsingular; if A possesses no inverse, it is called a singular matrix.
- If A is n x n, then $A^t$ must also be n x n.
- If A is invertible (has an inverse), then its inverse is unique.

**Rules**:
- If $A^{-1}A=I$ & $CA^{-1}=I \Rightarrow C=A$
- $(A^{-1})^{-1} = A$ [the inverse of the inverse is the original matrix]
- $(AB)^{-1} = B^{-1}A^{-1}$ [the inverse of a product is the product of the inverse in reverse order]
- $(A')^{-1} = (A^{-1})'$ [the inverse of the transpose is the transpose of the inverse]

## ■ The Inverse Matrix Using Determinants

$P_{99\text{-}103}$

$$A^{-1} = \frac{1}{|A|}.(A^{adj})'$$

**Rule**: An alternative method to calculate the inverse matrix is by using Determinant

## ■ Adjoint Matrix ($A^{adj}$)

**Rule**: **Adjoint Matrix** is obtained by taking the transpose of the cofactor matrix of [A].

## ■ The Cofactor Matrix

If $A_{3.3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

The cofactor of element a12 is: C12 = (-1) $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ = -(1) * $(a_{21} * a_{33} - a_{23} * a_{31})$

**Rule**: If A is a square matrix, (3 × 3) for example, then the minor of entry $a_{ij}$ is denoted by $M_{ij}$ and is defined to be the determinant of the submatrix that remains after the $i^{th}$ row and $j^{th}$ column are deleted from A.

## ■ The Inverse of a (2 × 2) Matrix

If $\boldsymbol{A_{2.2}} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{bmatrix}$, then

$$A^{-1}{}_{2.2} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

**Rule**: swap the positions of $a_{11}$ and $a_{22}$, put negatives in front of $a_{12}$ and $a_{21}$, and divide everything by the determinant $(a_{11}a_{22} - a_{12}a_{21})$

## ■ The Determinant of a (3 × 3) Matrix

If $A_{3.3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

$$|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

## ■ Cramer's Rule

P103-105

A rule that can be used to evaluate the endogenous variables- the vector X- (i.e., to solve systems of linear equations) without having to find the inverse of the coefficient matrix A.

**Cramer's Rule of (2x2) Matrix (Two Equations)**

If we are given a pair of simultaneous equations:

$$a_1 x + b_1 y = d_1$$

$$a_2 x + b_2 y = d_2$$

Then,

$$x = \frac{\begin{vmatrix} \boldsymbol{d_1} & b_1 \\ \boldsymbol{d_2} & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} a_1 & \boldsymbol{d_1} \\ a_2 & \boldsymbol{d_2} \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

**Note**: the j**th** column replaced by d

**Cramer's Rule of (3x3) Matrix (Three Equations)**

If we are given three of simultaneous equations:

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

Then,

$$x = \frac{\begin{vmatrix} \boldsymbol{d_1} & b_1 & c_1 \\ \boldsymbol{d_2} & b_2 & c_2 \\ \boldsymbol{d_3} & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} a_1 & \boldsymbol{d_1} & c_1 \\ a_2 & \boldsymbol{d_2} & c_2 \\ a_3 & \boldsymbol{d_3} & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}, \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & \boldsymbol{d_1} \\ a_2 & b_2 & \boldsymbol{d_2} \\ a_3 & b_3 & \boldsymbol{d_3} \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

**Note**: the j**th** column replaced by d

# Rules of Differentiation

## ■ Difference Quotient of a Function (Limit/ derivative)

P125-127

$$\frac{\boldsymbol{\Delta y}}{\boldsymbol{\Delta x}} = \frac{\boldsymbol{f(x_0 + \Delta x) - f(x_0)}}{\boldsymbol{\Delta x}}$$

If you have y=f(x) = ax$^{\beta}$-b, then

The difference quotient is $\frac{\Delta y}{\Delta x} = \boldsymbol{a\beta X_0} + \boldsymbol{a}\Delta \boldsymbol{X},$

The limit of the previous quotient is: $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (\boldsymbol{a\beta X_0} + \boldsymbol{a}\Delta \boldsymbol{X}) = \boldsymbol{a\beta X_0}$

The above quotient measures the average rate of change of y.

## Derivative and Functional Form

| Type of function | Form of function | Graph | Derivative Rule | Interpretation |
|---|---|---|---|---|
| y = constant | y = C | Horizontal line | $\frac{dy}{dx} = 0$ | Slope = 0 |
| y = linear function | y = ax + b | Straight line | $\frac{dy}{dx} = a$ | Slope = coefficient of x |
| y = polynomial of order 2 or higher | y = ax$^{n}$ + b | Nonlinear, one or more turning points | $\frac{dy}{dx} = anx^{n-1}$ | Derivative is a function; actual slope depends upon location (ie value of x) |

## Derivatives' Rules

- Constant Rule: f(x) = c then f ' (x) = 0
- Constant Multiple Rule: $g(x) = c \cdot f(x)$ then $g'(x) = c \cdot f'(x)$
- Power Rule: $f(x) = xn$ then $f'(x) = nx^{n-1}$
- Sum and Difference Rule: $h(x) = f(x) \pm g(x)$, then $h'(x) = f'(x) \pm g'(x)$
- Product Rule: $h(x) = f(x).g(x)$, then $h'(x) = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule: $h(x) = \frac{f(x)}{g(x)}, then\ h'(x) = \frac{f'(x).g(x) - f(x).g'(x)}{g(x)^2}$
- Chain Rule: $h(x) = f(g(x))$, then $h'(x) = f'(g(x))g'(x)$
- Logarithm Derivatives: $f(x) = \log_a(x)$, then $f'(x) = \frac{1}{\ln(a)x}$
- Logarithm Derivatives: $f(x) = ln(x)$, then $f'(x) = \frac{1}{x}$
- Logarithm Derivatives: $f(x) = \log_a(g(x))$, then $f'(x) = \frac{g'(x)}{\ln(a)g(x)}$

# Algebraic Rules

## The Quadratic Formula

Given a quadratic equation in the form:

$$aX^2 + bX + c = 0, \quad (a \neq 0)$$

Its two roots can be obtained from the quadratic formula:

$$\overline{X_1}, \overline{X_2} = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

## ■ Transforming Nonlinear Relation into Linear Relation by Taking Logarithm.

If $y = ax^b$, can be transformed into:

$$\log y = \log a = b \log x$$

# Leaflet- Economic Implication

## Economic Implication to Mathematical Tools

### ■ Domain and Range:

If $C = f(Q) = 150 + 7Q$ and if the firm has a capacity limit of 100 units of output per day.

**Then, the domain** = $\{Q|0 \leq Q \leq 100\}$; The **Range** = $\{C|150 \leq Q \leq 850\}$

### ■ Representing a Linear-Equation System using Matrices

A linear equation system can be written using matrix notation as follows:

$$\mathbf{AX} = \mathbf{d}$$

Where:

A: the coefficient matrix

X: the vector of endogenous variables

d: the vector of constant terms (and exogenous variables)

**Note:** The order here is very important; all endogenous variables are at the left side in the same order in all equations, while all constant terms and/or exogenous variables are at the right side of the 'equal' sign.

Note: A condition of finding a unique solution in the linear equation system

- The coefficient matrix (A) should be a square matrix
- The matrix (A) should be linearly independent in rows and columns (i.e., the determinant of A should not equal to zero).

### ■ Notes on Deriving a Marginal Function from a Total Function.

**Cost Functions:** The **total cost** is the cost of operating a business. Usually includes **fixed costs** and **variable costs**.

**Calculating Rate of Change of Cost Function?**

The **marginal cost function** is defined to be the derivative of the corresponding total cost function. If C(Q), Q is output, is the cost function, then C'(Q) is its marginal cost function. Thus, $MC = \frac{dC}{dQ}$

Notes:

- marginal = derivative
- Marginal costs can also be expressed as the change in the variable cost with respect to the output, that is: $MC = \frac{dVC}{dQ}$

- Marginal costs can also be expressed as the cost per unit of labor divided by the marginal product of labor, that is: $MC = \frac{w}{MPL}$, $MPL = \frac{\Delta Q}{\Delta L}$

# Economic Models

## ■ Economic Model Structure

1. **A model description:**
   i. **Identify** the dependent variable/s: the variable/s you want the model to calculate
   ii. **Identify** the independent variables: the variables you think will affect the dependent variable/s
   iii. **Find** all model assumptions and restricting conditions.
2. **Write down** the balance equations: You may want to have your old economic text books when you start writing the equations. Note that You must have as many equations as you have dependent variables (i.e., No more and no less).

   **Notes: you need to arrange** all the equations in the previous step carefully. When you have arranged the equations, you can think about the mathematics of solving them.

3. **Identify** endogenous and exogenous variables.
4. **Classify** the type of your model (**Static vs. dynamic**): If there are differential equations in your model, then your model is a "dynamic model". If there are no differential equations. then you have a "static model".
5. **Think** about the mathematical tools to find a solution to an economic model.
6. **Check** the solution and **set additional model restrictions, if exist,** to confirm that it gives sensible results.

## ■ Partial Market Equilibrium (A Good Market): A Linear Model

1. **A model description:**
   - It is a model of price determination in an isolated market.
   - One commodity is being considered.
   - Three variables are included in the model: quantity demanded $Q_d$, the quantity supplied $Q_s$, the commodity's price (P).
   - $Q_d$ is assumed to be a decreasing linear function of P [the law of demand] with a positive vertical intercept.
   - $Q_s$ is assumed to be an increasing linear function of P [the law of supply] with a negative vertical intercept, **why**? (to force the supply curve to have a positive horizontal intercept at P1, thereby satisfying the condition that producers will not supply unless the price is positive and sufficiently high.
   - The standard assumption is that equilibrium is obtained in the market if and only if the excess demand is zero ($Q_d - Q_s = 0$). [i.e., the market is cleared]
2. **The model balance equations are:**
   - $Q_d = a - bP,\ (a, b > 0)$→ a behavioral equation
   - $Q_s = -c + dP,\ (c, d > 0)$ → a behavioral equation

– $Q_d = Q_s$ → equilibrium condition

3. **Endogenous variables are: $\boldsymbol{Q_d, Q_s, P}$**
4. **The model is static**
5. **The solution:**
   - ✓ We need a solution to $\bar{Q}, \bar{P}$. You should note that all solution values should be expressed in terms of parameters (which represent given data for the model).
   - ✓ Ways of finding a solution:

**a) Solution by Elimination of Variables through substitution:**

From the above equations, we can find that:

$a - bP = -c + dP$, then

$\bar{P} = \frac{a+c}{b+d}$, then

$$\bar{Q} = a - b\frac{a+c}{b+d} = \frac{a(b+d)-b(a+c)}{b+d} = \frac{ad-bc}{b+d},$$

6. **Checking the solution and set additional model restrictions, if exist.**

✓ Since the denominator ($b+d$) is positive, the positive value of $\bar{Q}$ requires that the numerator ($ad$-$bc$) to be positive. Thus, the present model should contain additional restriction, that is $ad > bc$.

## ■ Partial Market Equilibrium (A Good Market): A Non-linear Model

1. **A model description:**
   - It is a model of price determination in an isolated market.
   - One commodity is being considered.
   - Three variables are included in the model: quantity demanded $Q_d$, the quantity supplied $Q_s$, the commodity's price (P).
   - $Q_d$ is assumed to be a decreasing non-linear function of P [let's assume a quadratic form] with a positive vertical intercept.
   - $Q_s$ is assumed to be an increasing linear function of P [the law of supply] with a negative vertical intercept, **why**?
   - The standard assumption is that equilibrium is obtained in the market if and only if the excess demand is zero ($Q_d - Q_s = 0$). [i.e., the market is cleared]
2. **The model balance equations are:**
   - $Q_d = a - bP^2,\ (a, b > 0)$ → a behavioral equation
   - $Q_s = -c + dP,\ (c, d > 0)$ → a behavioral equation
   - $Q_d = Q_s$ → equilibrium condition
3. **Endogenous variables are: $\boldsymbol{Q_d, Q_s, P}$**
4. **The model is static**
5. **The solution:**
   - ✓ We need a solution to $\bar{Q}, \bar{P}$.
   - ✓ Ways of finding a solution:

**a) Solution by Elimination of Variables through substitution:**

From the above equations, we can find that:

$a - bP^2 = -c + dP$, then

$P^2 + dP - c = 0$ → the quadratic equation is set to equal zero

Accordingly, we can expect to get two solutions to the above quadratic equation:

$$\overline{P_1}, \overline{P_2} = \frac{-d \pm \sqrt{(d^2 - 4c)}}{2}$$

## ■ General Market Equilibrium (A Good Market):(n- Commodity Case)

1. **A model description:**
   - Multiple commodities are being considered (complements and substitutes)
   - Three variables are included in the model: quantity demanded $Q_{di}$, the quantity supplied $Q_{si}$, the commodities' prices (P).
   - The standard assumption is that equilibrium is obtained in the market if and only if the excess demand is zero $(Q_{di} - Q_{si} = 0)$, $(i = 1,2, \dots \dots, n)$.
   - For simplicity, let's assume we have **<u>two-commodity market model</u>**.
   - $Q_{di}$ is assumed to be a decreasing linear function of P.
   - $Q_{si}$ is assumed to be an increasing linear function of P.
2. **The model balance equations are:**
   - $Q_{d1} = a_0 + a_1P_1 + a_2P_2,$ → a behavioral equation
   - $Q_{s1} = b_0 + b_1P_1 + b_2P_2,$ → a behavioral equation
   - $Q_{d1} = Q_{s1}$ → equilibrium condition
   - $Q_{d2} = \alpha_0 + \alpha_1P_1 + \alpha_2P_2,$ → a behavioral equation
   - $Q_{s2} = \beta_0 + \beta_1P_1 + \beta_2P_2,$ → a behavioral equation
   - $Q_{d2} = Q_{s2}$ → equilibrium condition
3. **Endogenous variables are: $\boldsymbol{Q_{di}, Q_{si}, P_i}$, $(\boldsymbol{i = 1, 2})$**
4. **The model is static**
5. **The solution:**
   - ✓ We need a solution to $\overline{Q_i}, \overline{P_i}$.
   - ✓ Ways of finding a solution:

**b) Solution by Elimination of Variables through substitution:**

$$a_0 + a_1P_1 + a_2P_2 = b_0 + b_1P_1 + b_2P_2$$
$$\alpha_0 + \alpha_1P_1 + \alpha_2P_2 = \beta_0 + \beta_1P_1 + \beta_2P_2,$$

**Thus,**

$$(a_0 - b_0) + (a_1 - b_1)P_1 + (a_2 - b_2)P_2 = 0$$
$$(\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)P_1 + (\alpha_2 - \beta_2)P_2 = 0,$$

**Lets define:**

$$\boldsymbol{C_i \equiv a_i - b_i}$$
$$\boldsymbol{\gamma_i \equiv \alpha_i - \beta_i}$$

**Thus,**

$$C_1P_1 + C_2P_2 = -C_0$$
$$\gamma_1P_1 + \gamma_2P_2 = -\gamma_0,$$

**Thus,**

$$\overline{P_1} = \frac{C_2\gamma_0 - C_0\gamma_2}{C_1\gamma_2 - C_2\gamma_1}$$

$$\bar{P}_2 = \frac{C_0\gamma_1 - C_1\gamma_0}{C_1\gamma_2 - C_2\gamma_1}$$

(from $\bar{P}_1$ and $\bar{P}_2$, we can substitute them and obtain $\bar{Q}_1$ and $\bar{Q}_2$).

**6. Checking the solution and set additional model restrictions, if exist.**

The present model should contain additional restriction, that is:

- Since division by Zero is undefined, the common denominator should be nonzero, that is $C_1\gamma_2 \neq C_2\gamma_1$.
- To assure a positive value of the price, the numerator should have the same sign as the denominator.

## ■ Two markets equilibrium Model

By Assuming perfectly competitive market: that is, both buyers and sellers are price-takers. Two goods (coffee and tea). Two goods are substitutable (not complementary). Each producer can produce only one good (short-run).

| In **market-1**: we have the following equations: | In **market-2**: we have the following equations: |
|---|---|
| $Q_{d1} = 10 - 2P_1 + P_{2;}$<br>$Q_{s1} = -2 + 3P_1;$<br>$Q_{d1} = Q_{s1}$ | $Q_{d2} = 15 + P_1 - P_{2;}$<br>$Q_{s2} = -1 + 2P_2;$<br>$Q_{d2} = Q_{s2}$ |
| Demand for good 1, $Q_{d1} = f(P_1, P_2)$<br>The coefficient of $P_1$ is negative due to the law of demand.<br>The coefficient of $P_2$ is positive due to the fact that the two goods are substitutes.<br>Producer produces only good 1 | Demand for good 2, $Q_{d2} = f(P_1, P_2)$<br>The coefficient of $P_2$ is negative due to the law of demand.<br>The coefficient of $P_1$ is positive due to the fact that the two goods are substitutes.<br>Producer produces only good 2 |

**Question**

How can we find the equilibrium prices and quantities for multiple market models?

**Solution:**

**Step1**: at equilibrium, equations can be rewritten as the following:

| | |
|---|---|
| ${Q_1}^* = 10 - 2P_1^* + P_2^*;$<br>${Q_1}^* = -2 + 3P_1^*;$ | ${Q_2}^* = 15 + P_1^* - P_2^*;$<br>${Q_2}^* = -1 + 2P_2^*;$ |

**Step2**: Using Matrices:

$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 1 & 0 & -3 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix} * \begin{bmatrix} {Q_1}^* \\ {Q_2}^* \\ P_1^* \\ P_2^* \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 15 \\ -1 \end{bmatrix}$$

All Coefficients of Endogenous Variables

All Constants

**Or $AX = B$**

**Coefficient matrix * Endogenous Variable Vector= Constant Vector**

The above matrix is a representation of the following system of linear equations:

$$1xQ_1^* + 0xQ_2^* + 2P_1^* - 1xP_2^* = 10 \quad \rightarrow \text{Eq (1)}$$
$$1xQ_1^* + 0xQ_2^* - 3P_1^* + 0xP_2^* = -2 \quad \rightarrow \text{Eq (2)}$$
$$0xQ_1^* + 1xQ_2^* - 1xP_1^* + 1xP_2^* = 15 \quad \rightarrow \text{Eq (3)}$$
$$0xQ_1^* + 1xQ_2^* + 0xP_1^* - 2P_2^* = -1 \quad \rightarrow \text{Eq (4)}$$

**Step 3: The Solution**

$$\begin{bmatrix} Q_1^* \\ Q_2^* \\ P_1^* \\ P_2^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 1 & 0 & -3 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -2 \end{bmatrix}^{-1} * \begin{bmatrix} 10 \\ -2 \\ 15 \\ -1 \end{bmatrix}$$

**Or $X = A^{-1}B$**

**Endogenous Variable Vector= the inverse of Coefficient matrix * Constant Vector**

## ■ National Income Equilibrium

1. **A model description:**
   - Assume a simple Keynesian national-income model,
   - Three variables are included in the model: National income (Y); Consumption (C); Investment ($I_0$); Government Expenditure ($G_0$);
   - The consumption consists of two parts: autonomous part and induced part.
2. **The model balance equations are:**

   $Y = C + I_0 + G_0$ → the equilibrium condition

   $C = a + bY \quad (a > 0; 0 < b < 1)$→ the behavioral equation

3. **Endogenous variables are: $Y, C$; while we have exogenous variables: $I_0, G_0$**
4. **The model is static**
5. **The solution:**
   - ✓ We need a solution to ***Y*** and ***C***.
   - ✓ Ways of finding a solution:

a) **Solution by Elimination of Variables through substitution:**

$Y = a + bY + I_0 + G_0$, or

$(1-b)Y = a + I_0 + G_0$, Thus,

$$\bar{Y} = \frac{a + I_0 + G_0}{(1-b)}$$

$$\bar{C} = a + b\frac{a + I_0 + G_0}{(1-b)} = \frac{a + b(I_0 + G_0)}{1-b}$$

**Note**: the solution value should be expressed only in terms of the parameters and exogenous variables.

6. **Checking the solution and set additional model restrictions, if exist.**
   - ✓ Both $\bar{Y}$ ***and*** $\bar{C}$ have the expression *(1-b)* in the denominator; thus, the present model should contain additional restriction, that is: $b \neq 1$;

- ✓ Both $\bar{Y}$ *and* $\bar{C}$ should be positive; thus, the present model should contain additional restriction, that is $I_0, G_0, > 0,\ a > 0$

- **Solution by Using Matrices:**

A simple national-income model can be written in matrix notation as

A X = D

$$A = \begin{bmatrix} 1 & -1 \\ -b & 1 \end{bmatrix}, X = \begin{bmatrix} Y \\ C \end{bmatrix}, and\ D = \begin{bmatrix} I_0 + G_0 \\ a \end{bmatrix}$$

The solution of the model is $X^* = A^{-1}D$

Or

$$\begin{bmatrix} Y^* \\ C^* \end{bmatrix} = \frac{1}{1-b} \begin{bmatrix} 1 & 1 \\ b & 1 \end{bmatrix} \begin{bmatrix} I_0 + G_0 \\ a \end{bmatrix} = \frac{1}{1-b} \begin{bmatrix} I_0 + G_0 + a \\ b(I_0 + G_0) + a \end{bmatrix}$$

- **Solving the National Income Equilibrium Model Using Cramer's Rule.**

A simple national-income model can be written in matrix notation as

$$Y - C = I_0 + G_0$$

$$-bY + C = a$$

$$Y^* = \frac{\begin{vmatrix} (I_0 + G_0) & -1 \\ a & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -b & 1 \end{vmatrix}} = \frac{I_0 + G_0 + a}{1 - b}$$

$$C^* = \frac{\begin{vmatrix} 1 & (I_0 + G_0) \\ -b & a \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -b & 1 \end{vmatrix}} = \frac{a + b(I_0 + G_0)}{1 - b}$$

## Important Economic Equations:

| Equation | Meaning |
|---|---|
| $\boldsymbol{Y \equiv C + I + X - N}$ | National Income Identity |
| $\boldsymbol{\pi \equiv R - C}$ | Total Profit Identity |
| $\boldsymbol{C = f(L, K) = C_0 + wL + vK}$ | Cost equation |
| $\boldsymbol{R = f(Q) = P * Q}$ | Revenue equation |
| $\boldsymbol{Q_d = Q_s}$ | Quantity demand = Quantity supply<br>The equilibrium condition in the good market |
| $\boldsymbol{S = I}$ | Intended saving = Intended investment<br>The equilibrium condition of the national income model. |
| $\mathbf{X^* = dX_1(P_{X1}, P_{X2}, m)}$ | Marshallian Demand ($dX_1$) functional form. |
| $\mathbf{X^* = hX_1(P_{X1}, P_{X2}, U)}$ | Hicksian Demand ($hX_1$) functional form. |

***The END***