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Leaflet- Economic Implication

Economic Implication to Mathematical Tools

■ Question 1: (The Relationship between MC & AC)

If you have the following cost function:

$$C = C(Q)$$

a) Find the marginal cost
b) Find the relationship between MC & AC, when AC is increasing, reaches minimum, or decreasing.

Solution:

a) The marginal cost equals: $\frac{dC}{dQ} = C'(Q)$
b) We need to find the slope (the partial derivative) of the AC function $AC = \frac{C(Q)}{Q}$,

$$\frac{dAC}{dQ} = \frac{Q.C'(Q)\text{-}C(Q)}{Q^2} = \frac{1}{Q}\left[C'(Q)\text{-}\frac{C(Q)}{Q}\right]$$

Thus,

$C'(Q) > \frac{C(Q)}{Q}$, if $\frac{dAC}{dQ} > 0$

$C'(Q) < \frac{C(Q)}{Q}$, if $\frac{dAC}{dQ} < 0$,

$C'(Q) = \frac{C(Q)}{Q}$, if $\frac{dAC}{dQ} = 0$

■ Question 2: (The Total Cost Function):

If you have the following total cost function:

$$C = Q^3 - 12Q^2 + 60Q$$

a) Find the marginal and average functions.
b) Graph the results.

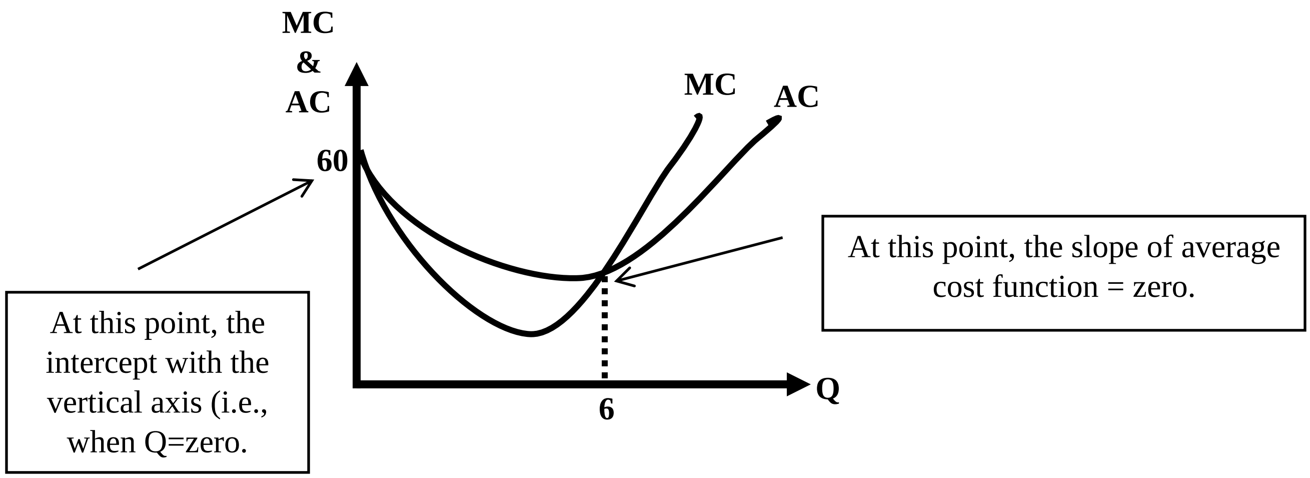
Solution:

a) The marginal cost function (MC)

$$MC = \frac{dC}{dQ} = 2Q^2 - 24Q \quad + 60$$

The average cost function $AC = \frac{C(Q)}{Q} = \frac{Q^3 - 12Q^2 + 60Q}{Q} = Q^2 - 12Q \quad + 60$

b) Graphing the results.



■ Question 3: (The Average Revenue Function):

If you have the following average revenue function, $AR = F(Q)$, prove that: the difference between MR & AR will always be $Q.\ F'\ (Q)$

Solution:

Marginal revenue function is derived from the total revenue function:

$$\text{TR} = \text{Q}*AR = \text{Q}*F(Q),$$

$$\text{MR} = \frac{\text{dTR}}{\text{dQ}} = \text{Q.F}'(\text{Q}) + \text{F}(\text{Q})$$

$$MR - AR = \text{Q.F}'(\text{Q}) + \text{F}(\text{Q})\text{-}F(Q)$$

$$MR - AR = \text{Q.F}'(\text{Q})$$

■ Question 4: (The Average Cost Function):

Given the following average-cost function:

$$AC = Q^2 - 4Q + 174,$$

a) Find the MC function
b) Is the given function more appropriate as a long-run or a short-run function? Why?

Solution:

a) MC is derived form the total cost function

$$TC = Q.AC = Q^3 - 4Q^2 + 174Q$$

$$MC = \frac{\text{dTC}}{\text{dQ}} = 3\text{Q}^2\text{-8Q} + 174$$

b) The given function is appropriate to the long-run, since there is no fixed cost (as shown in the total cost function)

■ Question 5: (The Average Revenue Function)

Given the following average revenue function: $AR = 15 - Q,$

Find the marginal revenue.

Solution:

$MR = \frac{\text{dTR}}{\text{dQ}} = 15\text{-}2Q$

■ Question 6: (The Total Revenue Function)

Given a total revenue function of a firm. $R = F(Q)$ & $Q = g\ (L)$,

Derive the marginal revenue product of labor by using the chain rule.

Solution:

Marginal Revenue Product of Labor (MRPL) is the extra revenue generated when an additional worker is employed.

MRPL = marginal product of labor x marginal revenue

$$\boldsymbol{MRPL} = \frac{\text{dR}}{\text{dQ}} * \frac{\text{dQ}}{\text{dL}} = F'(Q) * g'(Q)$$

■ Question 7: (The Total Revenue Function)

If you have the following revenue function, $R = 30Q - 2Q^2$, and the following short-run production function, $Q = 20L^2$, find the marginal product of labor (MRPL).

Solution:

MRPL = marginal product of labor x marginal revenue

$$\boldsymbol{MRPL} = \frac{\text{dR}}{\text{dQ}} * \frac{\text{dQ}}{\text{dL}} = F'(Q) * g'(Q)$$

$$F'(Q) = 30\text{-}4Q$$

$$g'(Q) = 40L$$

$$\boldsymbol{MRPL} = (30\text{-}4Q) * 40L = (1200L\text{-}3200L^3)$$

■ Question 8: (The Short-Run Production Function)

Consider the following short-run production function: $Q = 6L^2 - 0.4L^3$, find the value of (L) that maximizes output.

Solution:

The value of (L) that maximizes output is the value of L that is obtained after equating the first derivatives with zero:

$\frac{dQ}{dL} = 12L\text{-}1.2L^2 = 0$ (1),

The solutions for equation (1) is L=0 (rejected); or L=10 (accepted)

■ Question 9: (A Market Model- Comparative Static Analysis)

If you have the following demand and supply equations:

$$Q_d = a - bP \ \ (a, b > 0),$$

$$Q_s = -c + dP \ \ (c, d > 0)$$

Find the effect of a change in the slopes and intercepts on the equilibrium price.

Solution:

At equilibrium, $Q_d = Q_s$, $a - bP = -c + dP$,

$$P^* = \frac{a + c}{b + d}, \qquad Q^* = \frac{ad\text{-}bc}{b + d}$$

By applying the quotient rule:

$\frac{\partial P^*}{\partial a} = \frac{1}{b+d}$, $\frac{\partial P^*}{\partial c} = \frac{1}{b+d}$

$\frac{\partial P^*}{\partial b} = \frac{-(a+c)}{(b+d)^2}$, $\frac{\partial P^*}{\partial d} = \frac{-(a+c)}{(b+d)^2}$

Thus,

$$\frac{\partial P^*}{\partial a} = \frac{\partial P^*}{\partial c} > 0$$

$$\frac{\partial P^*}{\partial b} = \frac{\partial P^*}{\partial d} < 0$$

■ Question 10: (Inverse Demand and Marginal Revenue)

If the linear inverse demand function is P = 100-2Q, what is the marginal revenue function?

Solution:

The linear inverse demand function is:

$$\boldsymbol{P = 100 - 2Q,}$$

$$\textbf{\textit{Total revenue TR}} = \boldsymbol{P * Q = (100 - 2Q) * Q = 100Q - 2Q^2}$$

$$\boldsymbol{MR} = \frac{\mathrm{dTR}}{\mathrm{dQ}} = 100\text{-}4\mathrm{Q}$$

■ Question 11: (National Income and Multipliers- Comparative Statics)

Consider the following national income identity:

$Y = C + I_0 + G_0$

Where: $C = \propto + \beta(\mathrm{Y} - t); T = \gamma + \delta Y$

$$\propto > 0;\ \gamma > 0;\ 0 < \beta < 0;\ 0 < \delta < 0$$

a) Find the equilibrium national income (Y*).
b) Find the government expenditure multiplier?
c) Find the non-income tax multiplier.
d) Find the income-tax rate multiplier.

Solution:

a) The equilibrium national income (Y*).

$$Y^* = \frac{\propto\text{-}\beta\delta + I_0 + G_0}{1\text{-}\beta + \beta\delta}$$ → this is the equilibrium national income in its reduced form.

b) The government expenditure multiplier: $\frac{\partial \mathrm{Y}^*}{\partial \mathrm{G}_0}$

A multiplier is a quantity by which a given number is to be multiplied.

$$\frac{\partial \mathrm{Y}^*}{\partial \mathrm{G}_0} = \frac{1}{1\text{-}\beta + \beta\delta} > 0$$

c) The non-income tax multiplier: $\frac{\partial \mathrm{Y}^*}{\partial \gamma}$

$$\frac{\partial \mathrm{Y}^*}{\partial \gamma} = \frac{\text{-}\beta}{1\text{-}\beta + \beta\delta} < 0$$

d) Find the income-tax rate multiplier:

$$\frac{\partial Y^*}{\partial \delta} = \frac{-\beta(\propto -\beta\delta + I_0 + G_0)}{(1-\beta + \beta\delta)^2} = \frac{-\beta Y^*}{1-\beta + \beta\delta} < 0$$

Note: all the parameters $(\propto; \gamma; \beta; \delta)$ are non-negative. γ is positive, because even if Y is zero, the government will still have a positive tax revenue from tax bases other than income.

■ Question 12: (Jacobian Determinant)

The function f : R2 → R2 is given by:

$$F\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} x^2 - y^2 \\ x^2 + y^2 \end{pmatrix}$$

Where is f invertible? Find the Jacobian matrix of f^{-1} where f is invertible.

Solution:

$$JF = \begin{pmatrix} 2x & -2y \\ 2x & 2y \end{pmatrix},$$

$$\text{Det of JF} = 8xy$$

$$Jf^{-1} = \begin{pmatrix} 2x & -2y \\ 2x & 2y \end{pmatrix}^{-1} = \frac{1}{8xy}\begin{pmatrix} 2y & 2y \\ -2x & 2x \end{pmatrix}$$

■ Question 13: (Jacobian Determinant)

Find the Jacobian of the following two equations:

$$X = u^2 - v^2$$

$$Y = u^2 + v^2$$

Solution:

$$J = \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ 2u & 2v \end{vmatrix}$$

$$J = 2u*2v-[-2v*2u] = 8uv$$

■ Question 14: (Jacobian Determinant)

Use the Jacobian determinants to test the existence of functional dependence between the following paired functions:

$$Y_1 = 2x_1 + 3x_1$$

$$Y_2 = 4{x_1}^2 + 12x_1x_2 + 9{x_2}^2$$

Solution:

$$J = \begin{vmatrix} \frac{dy_1}{dx_1} & \frac{dy_1}{dx_2} \\ \frac{dy_2}{dx_1} & \frac{dy_2}{dx_2} \end{vmatrix}$$

$$|J| = \begin{vmatrix} 2 & 3 \\ 8x_1 + 12x_2 & 12x_1 + 18x_2 \end{vmatrix}$$

$$|J| = 24x_1 + 36x_2\text{-}(24x_1 + 36x_2) = 0$$

Thus, there exists a functional dependence

■ Question 15: (The Gradient Vector)

Write the gradients of the following functions:

a) $f(x, y, z) = x^2 + y^3 + z^4$
b) $f(x, y, z) = \text{xyz}$

Solution:

A gradient vector is a vector which lists all the partial derivatives (slopes) of a function.

a) $\text{grad } f(x, y, z) = \begin{vmatrix} 2x \\ 3y^2 \\ 4z^3 \end{vmatrix}$

b) $\text{grad } f(x, y, z) = \begin{vmatrix} yz \\ xz \\ xy \end{vmatrix}$

■ Question 16: (The Inverse Function Rule)

check whether the following equation is strictly increasing or decreasing and whether the inverse exists:

$$Y = 5X + 25$$

Solution:

$$\frac{dy}{dx} = 5$$

Thus, it is strictly increasing function, (i.e., positive regardless of the value of x),

Notes: inverse function rules:

- The function has to be one-to-one (injective). That is, for every *x*-value, there's got to be a unique *y*-value.
- Strictly monotonic functions are injective.

■ Question 17: (The Inverse Function Rule)

Regarding the following functions:

a) $y = -x^6 + 5$, $(x > 0)$
b) $y = 4x^5 + x^3 + 3x$

Are they strictly monotonic?

For each strictly monotonic function, find $\frac{dx}{dy}$ by the inverse function rule?

Solution:

a) since, x>0, then we have $\frac{dy}{dx} = -6x^5 < 0$
thus, the function is strictly decreasing

$$\frac{dx}{dy} = -\frac{1}{6x^5} \text{ (the reciprocal)}$$

b) $\frac{dy}{dx} = 20x^4 + 3x^2 + 3 > 0$

For any value of x, the function is strictly increasing

$$\frac{dx}{dy} = \frac{1}{20x^4 + 3x^2 + 3} \text{ (the reciprocal)}$$

■ Question 18: (The Total Derivatives)

Suppose we have a revenue function: $\boldsymbol{R} = \boldsymbol{P} * \boldsymbol{Y}$ where P is price and Y is output and is a function of Price and Cost, $Y = Y(P, C)$. What is the total differential of R with respect to Price and Cost?

Solution:

$$dR = \frac{dR}{dP}.dP + \frac{dR}{dC}.dC$$

■ Question 19: (The Total Derivatives-Cobb-Douglas)

If you have the following Cobb-Douglas preferences function:

$$U = X^{0.5}Z^{0.5}$$

Find the total differential of the Cobb-Douglas preferences function.

Solution:

$$du = U^{'}(x,z)\ dx\ + U^{'}(x,z)\ dz$$

$$du = 0.5\frac{z^{0.5}}{x^{0.5}}\ dx + \frac{x^{0.5}}{z^{0.5}} dz$$

■ Question 20: (Marginal Functions and Average Functions)

If you have the following demand function:

$$Q = 100 - 2P$$

a) Find the marginal function
b) Find the average function
c) Find the elasticity when P=25

Solution:

a) The marginal function

$$\frac{dQ}{dP} = -2$$

b) The average function

$$\frac{Q}{P} = \frac{100\text{-}2P}{P}$$

c) The elasticity when P=25

$$\varepsilon_d = \frac{\frac{\partial Q}{\partial P}}{\frac{Q}{P}} = \frac{-P}{50-P} = -1 \text{ (unitary)}$$

■ Question 21: (Marginal Functions and Average Functions)

If you have the following saving function:

$$S = S(y, i)$$

Find the partial elasticities.

$$\varepsilon_{\text{sy}} = \frac{\text{Marginal Funciton}}{\text{Average Function}} = \frac{\frac{\partial \text{S}}{\partial \text{y}}}{\frac{\text{s}}{\text{y}}}$$

$$\varepsilon_{\text{si}} = \frac{\text{Marginal Funciton}}{\text{Average Function}} = \frac{\frac{\partial \text{S}}{\partial \text{i}}}{\frac{\text{s}}{\text{i}}}$$

■ Question 22: (Total Differential of the Saving Function)

If you have the following saving function:

$$S = -20 + 0.2\,y + 0.1\,i$$

a) What is the value of autonomous savings?
b) What is the value of income-induced saving when income = 20 million?
c) What are the values of partial elasticities when S=2 & y=20, i = 0.2
d) Find the total differential of the saving function.

Solution:

a) The value of autonomous savings is -20
b) The value of income-induced saving when income = 20 million is:

0.25Y = 0.25*20 = 5 million

c) The values of partial elasticities when S=2 & y=20, i = 0.2 is:

$$\varepsilon_{\text{sy}} = \frac{\frac{\partial \text{S}}{\partial \text{y}}}{\frac{\text{s}}{\text{y}}} = 0.2 * \frac{20}{2} = 2$$

$$\varepsilon_{\text{si}} = \frac{\frac{\partial \text{S}}{\partial \text{i}}}{\frac{\text{s}}{\text{i}}} = 0.1 * \frac{0.2}{2} = 0.01$$

d) The total differential of the saving function:

$$\text{ds} = \frac{\partial \text{S}}{\partial \text{k}}.\text{dk} + \frac{\partial \text{S}}{\partial \text{y}}.\text{dy} + \frac{\partial \text{S}}{\partial \text{i}}.\text{di}$$

$$\text{ds} = 0.2\ \text{dy} + 0.1\ \text{di}$$

■ Question 23: (Direct Effect and Indirect Effect)

Let $Q = L^{\alpha} K^{\beta} \mathrm{T}^{\gamma}$ be a Cobb-Douglas production function where L represents labor, K represents Capital, T represents time.

If K and L can change over time, that is, the case that $L = L(t) = 20 + 0.5T$ and $K = K(t) = 15 + 2T$. Find the total derivative of this production function and indicate the value of direct and indirect effects regarding Time.

Solution:

$$\mathrm{dQ} = \frac{\partial \mathrm{Q}}{\partial \mathrm{L}}.\mathrm{dL} + \frac{\partial \mathrm{Q}}{\partial \mathrm{K}}.\mathrm{dK} + \frac{\partial \mathrm{Q}}{\partial \mathrm{T}}.\mathrm{dT}$$

$$\frac{\mathrm{dQ}}{\mathrm{dT}} = \frac{\partial \mathrm{Q}}{\partial \mathrm{L}}.\frac{\mathrm{dL}}{\mathrm{dT}} + \frac{\partial \mathrm{Q}}{\partial \mathrm{K}}.\frac{\mathrm{dK}}{\mathrm{dT}} + \frac{\partial \mathrm{Q}}{\partial \mathrm{T}}.\frac{\mathrm{dT}}{\mathrm{dT}}$$

$$\frac{\mathrm{dQ}}{\mathrm{dT}} = \frac{1}{2}\alpha \mathrm{L}^{\alpha\text{-}1} K^{\beta} \mathrm{T}^{\gamma} + 2\beta K^{\beta\text{-}1} L^{\alpha} \mathrm{T}^{\gamma} + L^{\alpha} K^{\beta} \gamma \mathrm{T}^{\gamma\text{-}1}$$

- The direct value is $L^{\alpha} K^{\beta} \gamma \mathrm{T}^{\gamma\text{-}1}$
- Indirect effect value is $\frac{1}{2}\alpha \mathrm{L}^{\alpha\text{-}1} K^{\beta} \mathrm{T}^{\gamma} + 2\beta K^{\beta\text{-}1} L^{\alpha} \mathrm{T}^{\gamma}$

The END